

Communication and Coordination: The Case of Boundedly Rational Players*

Tore Ellingsen[†]

Robert Östling[‡]

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Abstract

Using the level- k model of boundedly rational interaction, we fully characterize the effects of pre-play communication in symmetric and generic 2×2 games. We find that one-way communication weakly increases coordination on Nash equilibrium outcomes in all such games. Although one-way communication entails Nash equilibrium when relatively sophisticated players meet, there are games in which average payoffs fall when one-way communication is allowed. Two-way communication can yield higher average payoffs than one-way communication in coordination games such as Stag Hunt, but in other games two-way communication reduces both average payoffs and the degree of coordination below the no-communication level. Extending our analysis to larger and less symmetric games, we find that communication facilitates coordination in all two-player common interest games. However, we also identify games in which communication hampers coordination.

Keywords: Pre-play communication, coordination games, Stag Hunt, level- k , bounded rationality.

JEL codes: C72.

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[†]Address: Department of Economics, Stockholm School of Economics, Box 6501, SE-113 83 Stockholm, Sweden. E-mail: tore.ellingsen@hhs.se.

[‡]Address: Department of Economics, Stockholm School of Economics, Box 6501, SE-113 83 Stockholm, Sweden. E-mail: robert.ostling@hhs.se

1 Introduction

What is the role of non-binding and costless pre-play communication—that is, cheap talk—in games with complete information? According to Farrell (1987, 1988) and Rabin (1990, 1994), cheap talk can communicate intentions and thereby entails two primary benefits.¹ The first function is to *improve determinacy*. When the game has many efficient pure strategy equilibria, communication helps players coordinate partly (under multilateral communication) or fully (under unilateral communication) on some profile of efficient equilibrium actions. The second function is to *provide reassurance*. When an efficient equilibrium entails greater strategic risk than some inefficient outcome, communication helps to assure listeners about the speaker’s intention to behave in accordance with the efficient equilibrium.² Again, expected payoffs rise relative to the outcome without communication.

While theorists and experimentalists broadly agree that cheap talk can improve determinacy in mixed motive games like the Battle of the Sexes, they disagree about the extent to which cheap talk provides reassurance in coordination games like Stag Hunt. Notably, Aumann (1990) argues that cheap talk among rational players should not suffice to provide reassurance in the Stag Hunt game depicted in Figure 1.

	$H(\text{igh})$	$L(\text{ow})$
$H(\text{igh})$	9, 9	0, 8
$L(\text{ow})$	8, 0	7, 7

Figure 1. Stag Hunt

In this game, the two players both prefer the (H, H) equilibrium to the (L, L) equilibrium. Yet, without communication many theories predict the (L, L) equilibrium, since L is considerably less risky than H in case the player is uncertain about what the opponent will do. Farrell (1988) suggests that one-way communication suffices to solve the problem, because the message is *self-committing*. If sending the message “ H ” convinces the receiver that the sender intends to play H , the best response is for the receiver to play H , and thus the sender has an incentive to play according to the own message. Aumann (1990) objects that even a sender who has decided to play L has an incentive to induce the opponent to play H . That is, the message “ H ” is not *self-signaling*.

Farrell and Rabin (1996, page 114) acknowledge that communication in Stag Hunt may not work perfectly in theory, but they suggest that it will—at least to some extent—work in practice: “[A]lthough we see the force of Aumann’s argument, we suspect that cheap talk will do a good deal to bring Artemis and Calliope to the stag hunt.” Farrell and Rabin’s view

¹As emphasized by Myerson (1989) cheap talk can communicate both own intended actions (“promises”) and desires about others’ actions (“requests”). Like most of the literature, we focus on the former.

²For a non-technical introduction to the literature on cheap talk about intentions, see Farrell and Rabin (1996), especially pages 110–116.

bears a striking resemblance to the discussion of arms control talks in Schelling (1966, pages 260–264). Schelling too essentially argues that pre-play communication in Stag Hunt might improve coordination on the efficient outcome, albeit only imperfectly: “The hot line [between Moscow and Washington] is not a great idea, only a good one.”

The experimental evidence on behavior in Stag Hunt games is somewhat conflicting, but it strongly suggests that communication matters. For example, in an experiment by Charness (2000) one-way communication induces substantial coordination on the efficient equilibrium. In the prior experiment by Cooper, DeJong, Forsythe and Ross (1992) one-way communication is rather ineffective, whereas two-way communication often suffices to create efficient coordination. If the fully rational model cannot account for these patterns—or for the intuitions of Farrell, Rabin, and Schelling—is there any other model that can do so?

Crawford (2003) argues that communication frequently makes more sense if people are boundedly rational than if they are fully rational. His analysis considers a special class of zero-sum games, namely Hide and Seek games, with one-way communication. Our work adapts Crawford’s approach in order to study a different (and larger) class of games, while also considering a larger set of communication protocols.³ More precisely, we follow in Crawford’s footsteps by using the level- k model of bounded rationality, but we study both one-way and two-way pre-play communication, and we consider the class of all symmetric and generic 2×2 games. Besides coordination games like Stag Hunt and others, our analysis thus covers dominance solvable games (like the Prisoners’ Dilemma) and mixed motive games (like Chicken). The model’s predictions are broadly consistent with the available evidence and suggest several new avenues for empirical work.

The level- k model is a structural non-equilibrium model of initial responses that was introduced by Stahl and Wilson (1994, 1995) and Nagel (1995) and that has been shown to outperform equilibrium models in a range of one-shot games.⁴ The level- k model has the feature that players differ in the sophistication that they ascribe to their opponent. The most primitive player type that is assigned a positive probability in our model, the level-1 player, assumes that the opponent plays a random action (is “level-0”) and best responds given this belief. A level-2 player assumes that the opponent is a level-1 player, and so on.⁵ Like all level- k models, our model of pre-play communication is primarily a model of how people play a game the first time they play it. If a game is played repeatedly, players are likely to learn from previous rounds which raises a number of issues that are not captured in our model.

³Previously Cai and Wang (2006) have adapted Crawford’s model to study one-sided cheap talk in sender-receiver games. See also the ongoing work by Crawford (2007) and Wengström (2007).

⁴See for example Stahl and Wilson (1994, 1995), Nagel (1995), Costa-Gomes, Crawford and Broseta (2001), Camerer, Ho and Chong (2004), Costa-Gomes and Crawford (2006) and Crawford and Iriberry (2007) for various normal form game applications.

⁵A natural extension of the level- k model is to assume that a level- k player believes that the opponent is drawn from a distribution of more primitive player types; see Camerer et al. (2004) for an analysis of the ensuing cognitive hierarchy model. In the first version of our paper, Ellingsen and Östling (2006), we also considered the cognitive hierarchy model. Since the main insights are robust to the choice of model, we only develop the simple level- k model here.

For parameter choices that are typical in the level- k literature, our main results are the following: (i) One-way communication improves average payoffs in Stag Hunt games with a conflict between efficiency and strategic risk (such as that in Figure 1) and in some but not all mixed motive games. (ii) Two-way communication may yield higher average payoffs than one-way communication, but only in Stag Hunt games with a conflict between efficiency and strategic risk and in mixed motive games with high miscoordination payoffs. (iii) In mixed motive games with high miscoordination payoffs, average payoffs can be lower with communication than without. An additional remarkable finding is that if players are sufficiently sophisticated, both one-way and two-way communication suffices to attain the efficient outcome in Stag Hunt. This conclusion holds not only in the limit as sophistication goes to infinity; it suffices that both players perform at least two thinking steps.

A key to our results is that players are assumed to communicate their true intentions whenever they are indifferent between messages. In other words, they have a lexicographic preference for honesty, as in Demichelis and Weibull (2007). This tie-breaking assumption is innocuous enough from a psychological point of view, but has a powerful effect in our model. It directly implies that level-0 players are telling the truth (or more precisely, that level-1 players believe that their opponent will be honest). Even though we focus our analysis on the case in which there is actually never any level-0 player, the indirect impact on more advanced player types is significant: Level-0 behavior constitutes the level-1 player's model of the opponent. A level-1 receiver will thus play a best response to the received message. Since level-1 behavior constitutes the level-2 player's model of the world, a level-2 sender will therefore send a message that corresponds to the sender's favorite Nash equilibrium. Indeed, it is straightforward to check that all player types will communicate their intentions honestly under one-way communication. (However, this does not imply that one-way communication suffices to induce an efficient outcome. For example, in the Stag Hunt game above, level-1 players would send and play L .)

Extending our analysis to larger games and/or relaxing the symmetry assumption, we find that both one-way and two-way communication facilitates coordination in all two-player common interest games: When both players make at least two thinking steps, there is always coordination on (the best) Nash equilibrium in these games. This result is simple to prove, but nonetheless remarkable in view of the fact that coordination may require unrealistically many thinking steps when players cannot communicate.

On the other hand, we also identify games in which communication erodes coordination. The reason is that players have an incentive to deceive the opponent by misrepresenting their intentions. Even if the game has a unique pure strategy equilibrium, players can obtain large non-equilibrium payoffs if they successfully fool their opponent. When players are similar and not too sophisticated, they end up playing non-equilibrium strategies that may be either more or less profitable than the equilibrium.

Observe that we take for granted that players have access to a common language. That is, we take an eductive approach to communication. A substantial fraction of the literature on cheap talk starts from the presumption that messages are not inherently meaningful; instead,

messages may or may not acquire meaning in equilibrium—where equilibrium is typically depicted, implicitly or explicitly, as a steady state of an evolutionary process of random matches between boundedly rational players; see, for example, Matsui (1991), Wärneryd (1991), Kim and Sobel (1995), Anderlini (1999) and Banerjee and Weibull (2000). The eductive and evolutionary approaches are complementary, and our assumption of bounded rationality closes part of the gap between them. However, while the evolutionary approach can explain how language emerges in “old” games, the eductive approach asks how an existing language will be used in “new” games.

Within the evolutionary cheap talk literature, we are only aware of one contribution that emphasizes the distinction between one-way and two-way communication. In a paper quite closely related to ours, Blume (1998) proves that two-way communication can be superior to one-way communication in games with strategic risk, such as Stag Hunt. Interestingly, Blume’s result requires that messages have some small a priori information content. For example, players may have a slight preference for playing (H, H) if both players sent the message “ H ” and the expected payoffs to playing H and L are otherwise equal. As Blume notes, his assumption amounts to assuming some small amount of gullibility on the part of receivers. In our eductive model, honesty of level-0 senders is instead what drives the superiority of two-way communication in the Stag Hunt game. More recently, in a paper that is contemporaneous with ours, Demichelis and Weibull (2007) find that both one-way and two-way communication induces efficient equilibria in an evolutionary model when players have lexicographic preferences for truthfulness.

2 Model

Let G denote some symmetric and generic 2×2 game.⁶ The two strategies are labeled H and L . We refer to G as an *action game* and H and L as *actions*. In the action game G preceded by one-way communication, $\Gamma_I(G)$, one of the players is allowed to send one of two messages, h and l , before the action game G is played. These messages are assumed to articulate a statement about the sender’s intention (rather than for example a statement about which action the sender desires from the receiver). Nature decides with equal probability which of the players that acts as sender. We assume that players share a common language and that h corresponds to the intention to take action H and l to action L . Since the message is observed before the action game is played, the actions chosen by the receivers can be made conditional on the received message. A strategy s_i for a player i of the full game $\Gamma_I(G)$ prescribes what message m_i to send and action a_i to take in the sender role, and a mapping $f_i : \{h, l\} \rightarrow \{H, L\}$ from received messages to actions in the receiver role. We write a pure strategy of player i

⁶In a generic game, no player obtains exactly the same payoff for two different pure strategy profiles. We restrict attention to generic games merely in order to keep down the number of cases under consideration.

(given the received message m_j) as

$$s_i = \langle m_i, a_i, f_i(m_j = h), f_i(m_j = l) \rangle.$$

For example, $s_1 = \langle h, H, L, L \rangle$ means that player 1 sends the message h and takes the action H if he is the sender, while playing L whenever acting as receiver.

In the game with two-way communication, $\Gamma_{II}(G)$, both players simultaneously send a message $m_i \in \{h, l\}$ before the action game G is played.⁷ Since messages are observed before the action game G is played, the actions chosen can be made conditional on messages sent. A strategy s_i for player i of the full game is therefore given by a message m_i and a mapping $f_i : \{h, l\} \rightarrow \{H, L\}$ from the opponent's message to actions. A pure strategy of player i (given the message m_j sent by player j) can thus be written

$$s_i = \langle m_i, f_i(m_j = h), f_i(m_j = l) \rangle.$$

For example, $s_1 = \langle h, H, L \rangle$ means that player 1 sends the message h , but plays according to the received message (i.e., plays H if player 2 sends message h and L if player 2 plays message l).

Observe that we neglect unused strategy components. For example, we do not specify what action a player would take in the counterfactual case when he sends another message than the message specified by his strategy. The reason is that interesting counterfactuals cannot arise in our model, as will soon become clear.

Players' behavior depends on their degree of sophistication. A player of type 0 (or level-0), henceforth called a T_0 player, is assumed to understand only the set of strategies, and not how these strategies map into payoffs. Thus, T_0 makes a uniformly random action plan, sticking to this plan independently of any message from the opponent. (Hearing the opponent's intended action is of little help to a player who does not understand which game is being played.) Importantly, since T_0 players do not understand how their own or their opponent's actions map into payoffs, or how their messages may affect their opponent's action, they are indifferent concerning their own messages.

For positive integers k , a T_k player chooses a best response to (the behavior that the T_k player expects from) a T_{k-1} opponent. In particular, T_1 plays a best response to T_0 . When $k \geq 2$, T_k players will sometimes observe unexpected messages. In this case T_k assumes that the message comes from a T_{k-l} player, where $l \leq k$ is the smallest integer that makes T_k 's inference consistent. (As we shall see, T_0 sends all messages with positive probability, so $l \in \{1, \dots, k\}$ always exists.) Let p_k denote the proportion of type k in the player population. As we shall

⁷Simultaneous messages may appear to be an artificial assumption. However, besides preserving symmetry, the case of simultaneous messages may capture the notion from models with sequential communication that the first and the last speaker may both have an impact. At any rate, as Rabin (1994, page 390) has argued, the simultaneous communication assumption appears to put a useful lower bound on the amount of coordination that is attainable through cheap talk.

see, players who perform more than one thinking step often, but not always, behave alike. Therefore, it is convenient to let T_{k+} denote player types that perform at least k thinking steps.

When a player is indifferent about actions in G , we assume that the player randomizes uniformly. However, when the player is indifferent about what pre-play message to send, we assume that there is randomization only in case the player is unable to predict the own action—which can only happen under two-way communication. Otherwise, indifferent players send truthful messages (or more precisely, a message that conveys the action that the player expects to be playing). The assumption reflects the notion that people are somewhat averse to lying, but it does so without incurring the notational burden of introducing explicit lying costs into the model. (Our results are preserved under small positive costs of lying.) While such lexicographic preference for truthfulness is an apparently weak assumption, it has an immediate implication: The message by T_0 reveals the intended action. Or to put it even more starkly, T_1 believes in received messages. (In Section 2.3 we explore alternative assumptions regarding how T_0 treat messages.)

In Appendix 1 we explicitly characterize the strategies of all player types. However, it is common to argue that T_0 does not accurately describe the behavior of any significant portion of real adult people and that actual players are best described by a distribution with support only on T_1, T_2 and T_3 (e.g. Costa-Gomes et al. 2001 and Costa-Gomes and Crawford 2006). For some of our results we thus refer to type distributions consisting exclusively of players of these three types. To have shorthand definition, we say that $p = (p_1, p_2, \dots)$ is a *standard type distribution* if $p_k > 0$ for all $k \in \{1, 2, 3\}$ and $p_k = 0$ for all $k \notin \{1, 2, 3\}$.

2.1 Examples

Consider the Stag Hunt game in Figure 1. Absent communication, T_1 best responds to the uniformly randomizing T_0 by playing the risk dominant action L . Understanding this, the best response of T_2 is to play L as well. Indeed, by induction any player T_{1+} plays L . For standard type distributions with $p_0 = 0$, the unique outcome is the risk dominant equilibrium (L, L) .⁸ The level- k model hence provides a rationale for why players play the risk dominant equilibrium in coordination games without communication.

If players can communicate, one-way communication suffices to induce play of H by all types T_{2+} . The analysis starts by considering the behavior of T_0 (as imagined by T_1). By assumption, a T_0 sender randomizes uniformly over L and H , while sending the corresponding truthful message. A T_0 receiver randomizes uniformly over L and H . As a sender, T_1 best responds by playing the risk dominant action L , and sending the honest message l . As a receiver, T_1 believes that messages are honest and thus plays L following the message l and H following the message h . Consider now T_2 . A T_2 sender believes to be facing a T_1 receiver who

⁸Note that this is not about equilibrium selection in the ordinary sense. Players do not select among the set of equilibria, but best-respond to the behavior of lower-step thinkers. Their behavior ultimately results from the uniform randomization of T_0 , which explains the parallel to risk dominance.

best responds to the message, so T_2 sends h and plays H . A T_2 receiver, expects to receive an l message and therefore play L . If receiving a counterfactual h message, T_2 thinks it is sent by a truthful T_0 sender and therefore plays H . It is easily checked that all T_{2+} behave like T_2 , implying that there will be coordination on the payoff dominant equilibrium whenever two T_{2+} players meet and communicate. In other words, the level- k model not only shows that it is feasible for advanced players to coordinate on the payoff dominant equilibrium, but that the *unique* outcome is that they will do so. Note in particular how reassurance plays a crucial role in the example. When a receiver gets a message h , the receiver is reassured that the sender will play H , and is therefore also willing to play H . Even if the message h is actually only self-signaling for (the non-existing) level-0 senders, it is self-committing for all other types, and this suffices to attain efficient coordination as long as both parties perform at least two thinking steps.

In Stag Hunt, the reassurance role of communication is strengthened even more when both players send messages. Under such two-way communication, T_1 trusts the received message and responds optimally to it. Expecting to play either action with equal probability, T_1 sends both messages with equal probability. T_2 believes that the opponent listens to messages, and therefore sends h and plays H irrespective of the received message. T_{3+} players believe that the opponent will play H and they therefore play H and send an h message. If they receive an unexpected l message, they believe it comes from T_1 and therefore play H anyway (as T_1 will respond to the received h message by playing H). Note that under two-way communication, T_{2+} players are so certain that the opponent will play H that they play H irrespective of the received message.

Table 1 summarizes the action profiles that will result in the Stag Hunt under one-way and two-way communication. The notation $1S$ indicates a player of type 1 in the role of sender, and so on. “Uniform” indicates that all four outcomes are equally likely.

$\Gamma_I(G)$ (<i>one-way communication</i>)				$\Gamma_{II}(G)$ (<i>two-way communication</i>)			
	$0R$	$1R$	$\geq 2R$		0	1	≥ 2
$0S$	Uniform	$\frac{1}{2}HH, \frac{1}{2}LL$	$\frac{1}{2}HH, \frac{1}{2}LL$	0	Uniform	$\frac{1}{2}HH, \frac{1}{2}LL$	$\frac{1}{2}HH, \frac{1}{2}LH$
$1S$	$\frac{1}{2}LL, \frac{1}{2}LH$	LL	LL	1	$\frac{1}{2}HH, \frac{1}{2}LL$	Uniform	HH
$\geq 2S$	$\frac{1}{2}HH, \frac{1}{2}HL$	HH	HH	≥ 2	$\frac{1}{2}HH, \frac{1}{2}HL$	HH	HH

Table 1. Action profiles played in Stag Hunt with communication

Communication entails perfect coordination on the payoff dominant equilibrium whenever T_{2+} players meet. However, one-way and two-way communication differs in two respects whenever T_1 players are involved. With one-way communication, T_1 senders play L and the risk

dominant equilibrium therefore results whenever T_1 senders play (since T_0 does not exist). Under two-way communication, however, there is miscoordination in half of the cases when two T_1 players meet. Thus, there is a trade-off when choosing the optimal communication structure between coordination on either equilibria and achieving the payoff dominant equilibrium more often. For standard type distributions, two-way communication entails higher expected payoffs than one-way communication as long as $p_1 \in (0, 2/3)$.

In the Stag Hunt, communication increases players payoff because it brings sufficiently much reassurance for players to coordinate on the risky but payoff dominant equilibrium. In mixed motive games such as Battle of the Sexes and Chicken, communication instead serves the role of symmetry-breaking. To see this, consider the mixed motive game depicted in Figure 2, where $a < 3$ and $a \neq 2$. If $a = 0$, then this is a Battle of the Sexes, whereas it is a Chicken game if $a > 0$. The outcome for this game depends on whether L or H is the risk dominant action, i.e., whether $a \gtrless 2$. For simplicity, we disregard the possibility that $a = 2$, but allow the ‘‘Battle of the Sexes’’ possibility that $a = 0$ (although this makes the game non-generic).

	H	L
H	0, 0	3, 1
L	1, 3	a, a

Figure 2. Mixed motive game

First consider the case of no communication. T_1 then plays the risk dominant action, i.e., L if $a > 2$ and H if $a < 2$. T_2 responds optimally by playing H if $a > 2$ and L if $a < 2$. The behavior of more advanced players continues to alternate, odd types playing L if $a > 2$ and H otherwise, whereas even types play H if $a > 2$ and L otherwise. The outcome therefore depends on the type distribution, but there will generally be many instances of miscoordination.⁹

One-way communication powerfully breaks the symmetry inherent in such games with two pure asymmetric equilibria. If H is the risk dominant action, then T_{1+} senders send h and play H , whereas T_{1+} receivers optimally respond to messages. If instead L is risk dominant, a T_1 sender sends l and plays L , whereas T_{2+} senders continue to send h and play H . One-way communication therefore implies that T_{1+} players always coordinate on an equilibrium. Except in the case when L is risk dominant and the sender is of type T_1 , coordination is on the sender’s preferred equilibrium.

It is unsurprising that one-way communication can break the symmetry and achieve coordination in games with two asymmetric equilibria. However, our analysis also reveals the novel possibility that in some versions of Chicken some players propose and play their least favorite equilibrium. T_1 senders play their risk-dominant action which may not correspond to their preferred equilibrium, whereas T_2 senders are confident in reaching their preferred equilibrium. Table 2 shows the outcomes that will result without communication and with one-way

⁹The outcome without communication does generally not resemble the symmetric mixed strategy equilibrium, but may happen to do so for certain combinations of payoff configurations and type distributions.

communication, demonstrating the improved coordination on equilibrium outcomes.

G (no communication)				$\Gamma_I(G)$ (one-way communication)			
	0	Odd	Even		0R	1R	$\geq 2R$
0	Uniform	$\frac{1}{2}HL, \frac{1}{2}LL$	$\frac{1}{2}LH, \frac{1}{2}HH$	0S	Uniform	$\frac{1}{2}HL, \frac{1}{2}LH$	$\frac{1}{2}HL, \frac{1}{2}LH$
Odd	$\frac{1}{2}LH, \frac{1}{2}LL$	LL	LH	1S	$\frac{1}{2}LH, \frac{1}{2}LL$	LH	LH
Even	$\frac{1}{2}HL, \frac{1}{2}HH$	HL	HH	$\geq 2S$	$\frac{1}{2}HL, \frac{1}{2}HH$	HL	HL

Table 2. Action profiles played in mixed motive games ($a > 2$)

Although one-way communication entails more equilibrium coordination than no communication, more coordination need not raise players' average payoffs. If $a > 2$, then players prefer the (L, L) outcome to ending up in either equilibrium with equal probability. If the type distribution is such that the (L, L) outcome results sufficiently often without communication, average payoffs are thus higher without communication. For example, when $a = 5/2$ and there is a standard type distribution with $p_2 < 1/3$, then average payoffs are lower under one-way communication than under no communication.

Suppose players could choose whether to engage in communication or not, and suppose that the allocation of roles is random. Each player type k would then consider the own expected payoff in each regime conditional on meeting a player of type $k - 1$. To illustrate that players may prefer not to communicate, we consider the case when $a = 0$, i.e., the Battle of the Sexes. Absent communication, T_3 believes that the opponent will play L and thus obtains the preferred equilibrium payoff. With one-way communication and a random allocation of roles, however, T_3 expects to end up in either equilibrium with equal probability. That is, T_3 expects to be better off if communication is impossible.

2.2 Results

In this section we generalize the findings from the previous section to all symmetric and generic 2×2 games, disregarding (the measure zero class of) games in which neither action is risk dominant. There are three broad classes of such games. The first class of games are the dominance solvable ones, like Prisoners' Dilemma. We use the convention of labelling the dominant action of these games $H(igh)$. The second class are coordination games, where we follow the example above and label the actions corresponding to the payoff dominant equilibrium $H(igh)$. The third class of games are mixed motive games like the one in Figure 2. For this class of games, we label the action corresponding to a player's preferred equilibrium $H(igh)$. In Appendix 1, we completely characterize behavior of all player types $k \in \mathbb{N}$ for these three classes of games. These characterizations provide the foundation for the results in this section, where we focus

on average outcomes under standard type distributions.

Our first result states the conditions under which one-way communication serves to increase players' average payoffs relative to no communication.

Proposition 1 *Given a standard type distribution, the average payoff associated with $\Gamma_I(G)$ exceeds the average payoff associated with G if and only if (i) G is a coordination game with a conflict between risk and payoff dominance, or (ii) G is a mixed motive game that satisfies either*

a. L is risk dominant and

$$\left(\frac{1}{2} - p_2(1 - p_2)\right) (u_{HL} + u_{LH}) > p_2^2 u_{HH} + (1 - p_2)^2 u_{LL},$$

or

b. H is risk dominant and

$$\left(\frac{1}{2} - p_2(1 - p_2)\right) (u_{HL} + u_{LH}) > (1 - p_2)^2 u_{HH} + p_2^2 u_{LL}.$$

Proof. In Appendix 2. ■

If we replace p_2 by p_E , the probability that players think an even number of steps, Proposition 1 generalizes straightforwardly to all type distributions in which $p_0 = 0$. In our examples, we have already explained why one-way communication improves average payoffs in Stag Hunt, and indicated why it sometimes fails to improve payoffs in mixed motive games. A straightforward implication of Proposition 1 is that one-way communication raises the average payoff in the Battle of the Sexes. (To see this, recall that in Battle of the Sexes $0 = u_{HH} = u_{LL} < u_{LH} < u_{HL}$, which implies that H is risk dominant.) Proposition 1 also implies that communication does not improve average payoffs in dominance solvable games. For Chicken, the impact of communication hinges more delicately on parameters, and communication may even serve to reduce payoffs.

Corollary 1 *Given a standard type distribution, the average payoff associated with $\Gamma_I(G)$ is smaller than the average payoff of G if and only if G is a Chicken that satisfies either*

a. L is risk dominant and

$$\left(\frac{1}{2} - p_2(1 - p_2)\right) (u_{HL} + u_{LH}) < p_2^2 u_{HH} + (1 - p_2)^2 u_{LL},$$

or

b. H is risk dominant and

$$\left(\frac{1}{2} - p_2(1 - p_2)\right) (u_{HL} + u_{LH}) < (1 - p_2)^2 u_{HH} + p_2^2 u_{LL}.$$

Proof. In Appendix 2. ■

Since H is risk dominant in Battle of the Sexes, one-way communication suffices to attain perfect coordination on the speaker's preferred equilibrium outcome. Thus, we here have a case in which the prediction from the level- k model coincides with the prediction from fully rational models. Likewise, the ineffectiveness of cheap talk in dominance solvable games is the same as in the fully rational model. At a deeper level, the two approaches also share the property that communication, if anything, pulls players towards Nash equilibria.

Proposition 2 *For any distribution of types, the frequency of coordination on pure strategy Nash equilibrium profiles is weakly greater in $\Gamma_I(G)$ than in G .*

Proof. In Appendix 2. ■

The pull towards Nash equilibria is so strong that one-way communication results in equilibrium play whenever T_{1+} meet. Moreover, T_{2+} always play the action corresponding to the sender's preferred equilibrium.

Corollary 2 *For type distributions with $p_0 = 0$, players in $\Gamma_I(G)$ always coordinate on pure strategy Nash equilibrium profiles. If in addition $p_1 = 0$, players in $\Gamma_I(G)$ always coordinate on the sender's preferred equilibrium.*

Proof. Follows directly from Tables A1 to A4 in the proof of Proposition 2. ■

In contrast to one-way communication, two-way communication may destroy not only average payoffs but also coordination on equilibrium outcomes. For example, suppose there are only T_1 players and let G be a coordination game in which payoff and risk dominance coincide. Then $\Gamma_{II}(G)$ entails miscoordination in half of the cases, because T_1 sends random messages while listening to received messages. By contrast, in G and in $\Gamma_I(G)$ two T_1 players always play the (payoff and risk) dominant equilibrium. Our model therefore captures the intuition that two-way communication can bring noise in the form contradictory messages.

Nevertheless, there are important classes of games in which two-way communication outperforms one-way communication.

Proposition 3 *Given a standard type distribution, the average payoff associated with $\Gamma_{II}(G)$ exceeds the average payoff associated with $\Gamma_I(G)$ if and only if (i) G is a coordination game in which L is the risk dominant action and $(4 - 3p_1)u_{HH} + p_1(u_{LH} + u_{HL}) > (4 - p_1)u_{LL}$, or (ii) G is a mixed motive game with a type distribution satisfying the following condition:*

$$1 + \frac{2(p_1 - 1)(p_1 - 1 + 2p_3)}{p_1^2 + 4p_3^2} < \frac{u_{LL} - u_{HH}}{u_{LH} + u_{HL} - 2u_{HH}}.$$

Proof. In Appendix 2. ■

The Stag Hunt game in Figure 1 belongs to the first class of games identified by Proposition 3. For that particular game, two-way communication yields higher expected payoff than one-way communication whenever $p_1 \in (0, 2/3)$. The second class of games identified in Proposition 3 is harder to specify because of the cycling patterns of behavior under two-way communication in mixed motive games. However, for two-way communication to be beneficial, the payoff when both players play L must be sufficiently high (at least $(u_{HL} + u_{LH})/2$) and in addition the type distribution has to be such that the miscoordination outcome (L, L) happens sufficiently often with two-way communication. For example, with only type T_3 players, the outcome is (L, L) under two-way communication, whereas such players coordinate on an asymmetric equilibrium with one-way communication.

2.3 Robustness

How robust are our results to the assumptions that we have made about T_0 ? That is, could T_1 have some other plausible hypothesis concerning the opponent’s behavior?

One possible hypothesis is that T_0 behaves as above, except sending random messages. In this case communication ceases to have any effect whatsoever in our model: behavior is the same in $\Gamma_I(G)$, $\Gamma_{II}(G)$ and G . This specification is strongly at odds with the evidence that communication matters in many game experiments. In our view, the specification serves primarily to highlight the importance of our “weak preference for truthfulness” assumption.

Another hypothesis is that T_0 acts as above, except responding systematically to received messages. The idea is that if the actions of both players have the same label, the receiver could imitate (or differentiate) based on the sender’s message. The most natural way to account for such imitation is to allow heterogeneous T_1 players, some believing that receivers randomize, others believing that receivers imitate.¹⁰ With one-way communication, this implies that some T_1 players believe T_0 receivers randomize, whereas others believe that they imitate. With two-way communication, some T_1 players believe that opponents are truthful, whereas other believe they imitate. Let us now consider the consequences of this specification.

First consider the Stag Hunt in Figure 1. Under one-way communication, T_1 senders who believe that receivers imitate send the message h and play H . This in turn implies that T_2 receivers respond to messages as if they were truthful irrespective of which kind of T_1 sender they think they face. Under one-way communication, the only difference compared to our original

¹⁰An alternative is to let T_1 assume that some fraction of T_0 imitates rather than randomizes. In this case, T_1 is sophisticated enough to consider heterogeneity among T_0 . We do not think this is plausible, and the consequences are counterfactual too: Consider one-way communication in the Battle of the Sexes. If there is heterogeneity among T_0 , T_1 will send l and play H —believing that some opponents ignore their message, whereas others imitate their message and play L . Since p_1 is typically estimated to be quite high, the implication is that sending l and playing H would be a relatively common practice. Cooper, DeJong, Forsythe and Ross (1989) studies one-way communication in Battle of the Sexes. They find that only 2 percent of all senders even sent an l message.

assumption is that there will be somewhat more coordination on (H, H) since some T_1 senders now play H . Under two-way communication, T_1 players who believe that opponents imitate send h and play H instead of responding to received messages. T_2 players therefore optimally send h and play H irrespective of which type of T_1 player they meet. Since miscoordination only occurs whenever two T_1 players that send random messages meet, there will now be more equilibrium coordination compared to the standard case.

Second, consider one-way communication in the Battle of the Sexes. While T_1 receivers, and hence T_2 senders, behave as before, T_1 senders that believe they face imitators now send l and play H . In the previous footnote, we have already argued that this behavior is implausible and that the fraction of such T_1 players must therefore be small. However, irrespective of how small a proportion they constitute, T_2 receivers now play L irrespective of what message they receive. This implies that T_3 senders send h and play H . Under a standard type distribution, the outcome in terms of observed action profiles is thus the same as before.

Although some details of the analysis change with the introduction of heterogeneous T_1 players, we conclude that the main mechanisms are robust to this modification.

3 Extensions

So far, we have confined attention to 2×2 games. In principle it is straightforward to extend the analysis to games with more players and strategies.¹¹ In this section, we show that the reassurance property of communication extends to two-player games in which players' interests are sufficiently well aligned. When attractive non-equilibrium outcomes are present, however, senders might try obtain these by deceiving the opponent. The possibility of deception implies that one-way communication may hamper coordination on Nash equilibria.

3.1 Common interest games

The Stag Hunt example illustrates that pre-play communication facilitates the play of a risky payoff dominant equilibrium. Since our model does not assume equilibrium play, it is also applicable to situations in which players realistically fail to play a unique and efficient Nash equilibrium—such as the High Risk game, devised by Gilbert (1990) and reproduced in Figure 3 (in which best replies are marked with asterisks).¹² Absent communication, the level- k model predicts that two T_{5+} players coordinate on the unique pure strategy equilibrium (U, X) , whereas all less sophisticated players fail to do so.¹³ In contrast, one-way and two-way

¹¹We restrict attention to two-player games here, but the first version of this paper, Ellingsen and Östling (2006), contains some n -player games to which some of the main intuitions extend.

¹²Experimental results of Burton and Sefton (2004) confirm the prevalence of coordination failure in one-shot play of the High Risk game, but demonstrate that players learn to play the equilibrium after having played a number of practice rounds with the same opponent.

¹³To see this, note that T_1 plays W and Z since these are the risk dominant actions. Using the best responses indicated in Figure 3 it follows that T_2 plays V and X , T_3 plays U and Y , T_4 plays W and X , and finally that

communication implies that T_{2+} coordinate on equilibrium. That is, much less sophistication is required to reach equilibrium with communication than without.¹⁴

	X	Y	Z
U	$5^*, 5^*$	$-50, -50$	$2, 4$
V	$-50, -50$	$2, 4^*$	$4^*, 3$
W	$4, 4^*$	$3^*, 3$	$3, 3$

Figure 3. High Risk game

The positive effect of communication in the High Risk game extends to all finite and normal form two-player games which has a payoff dominant equilibrium that gives strictly higher payoffs to both players than all other outcomes of a game, i.e., to all *common interest games*. For this class of games it is straightforward to show that T_{2+} coordinate on the payoff dominant equilibrium. The underlying mechanism is that since T_1 listens and best responds to messages, T_2 can achieve the best possible outcome by sending and playing the payoff dominant equilibrium.

Proposition 4 *Let G be a two-player common interest game. For type distributions with $p_0 = p_1 = 0$, players in $\Gamma_I(G)$ and $\Gamma_{II}(G)$ always coordinate on the payoff dominant Nash equilibrium.*

Proof. First consider $\Gamma_I(G)$. A T_1 sender sends and plays the action that is optimal given that the opponent randomizes uniformly over actions. If there are several such actions, T_1 plays each of them with equal probability and sends a truthful message. As a receiver, T_1 best responds to messages. Since the payoff dominant equilibrium gives the highest possible payoff, T_2 sends and plays the corresponding action as sender, while best responding to messages as receiver. It follows that T_{3+} behaves as T_2 . Now consider $\Gamma_{II}(G)$. T_1 believes the opponent is truthful and therefore best responds to messages, but sends a random message. T_{2+} believes the opponent best responds and therefore sends and plays the payoff dominant equilibrium irrespective of the received message. ■

3.2 Other games

In common interest games and in symmetric 2×2 games with one-way communication, players always represent their intentions truthfully. In other classes of games, however, this is not

T_{5+} plays U and X .

¹⁴To see this, first consider one-way communication. A T_1 row sender sends w and plays W , while a column sender sends z and plays Z . A T_1 receiver best responds to messages. A T_{2+} row sender therefore sends u and plays U , while a column sender sends x and plays X , while a T_{2+} receiver best responds to messages. Now consider two-way communication. T_1 believes the opponent is truthful and therefore best responds to messages and randomize what message to send. A T_{2+} row player therefore sends u and plays U while a column player sends x and plays X .

necessarily the case. Crawford (2003) already shows how deception arises naturally in a level- k model of communication in Hide-and-Seek games. We observe that deception can also arise in an asymmetric dominance solvable 2×2 game with a unique pure strategy equilibrium. Consider the game in Figure 4.

	Y	Z
W	$3^*, 2^*$	$4^*, 0$
X	$0, 0$	$0, 1^*$

Figure 4. Asymmetric 2×2 game

The game's unique pure strategy equilibrium is (W, Y) . Since W and Y are the risk dominant actions, T_{1+} players coordinate on the (W, Y) equilibrium if they are not allowed to communicate. Now consider one-way communication. Suppose that the row player acts as sender and the column player acts as receiver. The T_1 sender sends w and plays W , while a T_1 receiver best responds to received messages. A T_2 sender therefore sends x , but plays W , while a T_2 receiver best responds to messages. T_3 sends x but plays W , while a T_3 receiver ignores messages and always plays Y . Whenever T_{3+} players meet, the resulting outcome is the sender's preferred equilibrium, but not when less sophisticated players play. In contrast to Proposition 2, one-way communication leads to less equilibrium coordination than no communication unless all players carry out three or more thinking steps.

Proposition 2 does not generalize to symmetric two-player games with more than two actions either. To see this, consider the game in Figure 5.¹⁵ This symmetric 3×3 game has a unique pure strategy equilibrium, (H, H) , for all $n > 1$, but the game also has the asymmetric outcomes (H, L) and (L, H) that are attractive either to the row or column player. Since there is a third strategy, D , which has L as its best response, some senders will try to use this strategy to deceive the other player into playing L .

	H	L	D
H	$4/n^*, 4/n^*$	$(4 + 1/n)^*, 0$	$0, 0$
L	$0, (4 + 1/n)^*$	$0, 0$	$1^*, 1$
D	$0, 0$	$1, 1^*$	$0, 0$

Figure 5. Symmetric 3×3 game

Specifically, consider the case when $n = 1$ and pre-play communication is not possible. In that case T_1 would play H since it is the best action to take if the opponent randomizes uniformly, and T_{2+} would best respond by playing H . One-way communication, however, makes it more difficult to reach equilibrium. A T_1 sender sends h and plays H , while a T_1 receiver best responds (as indicated by the asterisks in Figure 5) to the received message. A T_2 sender sends

¹⁵Although this game is non-generic, the below discussion does not hinge on that.

d , but plays H , while a T_2 receiver best responds to received messages. A T_3 sender sends d and plays H , while a T_3 receiver plays H irrespective of the received message. A T_{4+} sender is indifferent about what message to send and is thus truthful, sending h and playing H ; a T_{4+} receiver ignores messages and plays H . We conclude that T_{3+} coordinate on (H, H) and that one-way communication consequently lowers equilibrium coordination unless all players make three or more thinking steps.

A modification of the game illustrates how the number of thinking-steps required to reach equilibrium may increase linearly with the size of the game. Consider the $3N \times 3N$ game shown in Figure 6. It has the game in Figure 5 on the main diagonal and zero payoffs elsewhere.

	H_1	L_1	D_1	H_2	L_2	D_2	\dots	H_N	L_N	D_N
H_1	4, 4	5, 0	0, 0	0, 0	0, 0	0, 0	\dots	0, 0	0, 0	0, 0
L_1	0, 5	0, 0	1, 1	0, 0	0, 0	0, 0	\dots	0, 0	0, 0	0, 0
D_1	0, 0	1, 1	0, 0	0, 0	0, 0	0, 0	\dots	0, 0	0, 0	0, 0
H_2	0, 0	0, 0	0, 0	2, 2	4.5, 0	0, 0	\dots	0, 0	0, 0	0, 0
L_2	0, 0	0, 0	0, 0	0, 4.5	0, 0	1, 1	\dots	0, 0	0, 0	0, 0
D_2	0, 0	0, 0	0, 0	0, 0	1, 1	0, 0	\dots	0, 0	0, 0	0, 0
\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\ddots	0, 0	0, 0	0, 0
H_N	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	$\frac{4}{N}, \frac{4}{N}$	$4 + \frac{1}{N}, 0$	0, 0
L_N	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	$0, 4 + \frac{1}{N}$	0, 0	1, 1
D_N	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	1, 1	0, 0

Figure 6. Symmetric $3N \times 3N$ game

Let messages be denoted m_n , with $m \in \{h, l, d\}$ and $n \in \{1, 2, \dots, N\}$. Without communication, T_{1+} plays H_1 as in the 3×3 game. However, when one-way communication is allowed, all players must make at least $2N + 2$ thinking steps in order to coordinate on the unique equilibrium (H_1, H_1) . To see why, note first that T_1 through T_3 will behave as in the 3×3 game, but that receivers will best-respond to all messages m_n with $n \in \{2, 3, \dots, N\}$, believing those messages to come from T_0 . A T_4 sender therefore sends d_2 and plays H_2 in order to get the outcome (H_2, D_2) which is preferred over (H_1, H_1) . T_5 receivers do not believe in d_2 messages and therefore play H_2 if either h_2 , l_2 or d_2 is played. In turn, T_6 senders send d_3 and play H_3 in order to induce the (H_3, L_3) outcome. The inductive argument continues like this up until T_{2N+1} sends d_N and plays H_N . A T_{2N+2} sender cannot hope to get anything better than (H_1, H_1) and therefore sends h_1 and plays H_1 , whereas a T_{2N+2} receiver plays H_n whenever h_n , d_n or l_n is played (for all n).

This example illustrates that the degree of sophistication required to play equilibrium increases with the size of the game. Since the degree of sophistication required is unrealistically high, in these games players coordinate better if they are unable to communicate.

3.3 Other communication protocols

Like much of the cheap talk literature, we have here considered communication of intentions. Messages are of the form “I plan to play...”. What would happen if players communicated requests instead, that is if messages were of the form “I want you to play...”? While the model still admits a notion of truthfulness, the analysis would be quite different. For example, it is no longer clear that T_1 players should care about the messages that they receive, since T_0 players’ requests may reveal nothing about their intentions. We thus expect that credulity will play a more important role than truthfulness in this case. Specifically, communication might now affect behavior if T_1 senders believe that receivers are credulous in the sense that they fulfill requests. Preliminary investigations suggest that the ensuing analysis offers a perspective on how cheap talk may be used to understand cheating in games, but we leave a fuller analysis for a separate paper.

4 Evidence

The level- k model of pre-play communication is primarily a model to explain initial responses, i.e., the behavior of players that play a game for the first time. If players gain experience of the game and the population of players, they are likely to change their model of opponents’ behavior or perhaps think further and proceed to higher levels of reasoning. In experimental work on pre-play communication, players typically play the same game in several rounds. Strictly speaking, most of the available evidence is thus inadequate for our purposes. With this caveat in mind, let us briefly discuss some of the most relevant communication experiments.

Two papers contrast one-way and two-way communication in Stag Hunt games. Cooper et al. (1992) report that average coordination on the payoff dominant equilibrium is 0 percent without communication, 53 percent with one-way communication and 91 percent with two-way communication. This study therefore strongly suggests that communication plays a reassurance role.¹⁶ Burton, Loomes and Sefton (2005) on the other hand find that one-way communication results in 52 percent coordination on the payoff dominant equilibrium, whereas two-way communication led to average coordination on the payoff dominant equilibrium of only 34 percent. Both papers find that behavior varies substantially across sessions, indicating that heterogeneity in early rounds of the game affect players choices in later rounds.

In addition to these two studies, there is also a few studies of the Stag Hunt game that investigate either one-way or two-way communication. Duffy and Feltovich (2002) finds that one-way communication entails coordination on the payoff dominant equilibrium in 84 percent of the cases with one-way communication and in 61 percent of the cases without communication. Charness (2000) studies the effect of one-way communication in three versions of the Stag Hunt and finds 86 percent coordination on the payoff dominant equilibrium with one-way com-

¹⁶Relatedly, Ellingsen and Johannesson (2004) identifies a reassurance role of communication in hold-up games with multiple equilibria.

munication. Clark, Kay and Sefton (2001) study two-way communication in two different Stag Hunt games. In the first game, playing L yields the same payoff irrespective of the opponents behavior. In this game, coordination on the payoff dominant equilibrium is 2 percent without communication and 70 with two-way communication. In a standard Stag Hunt game, they find that coordination on the payoff dominant equilibrium occurs in only 19 percent of the cases with two-way communication.

It is difficult to draw clear conclusions regarding pre-play communication in the Stag Hunt based on these studies. The degree of coordination on the payoff dominant equilibrium varies greatly and does not seem to systematically depend on the communication technology. Our analysis suggests that the precise interpretation of messages in terms of intentions or requests as well as the composition of the player population might cause some of the differences, but the only reliable way to find out is to conduct new experiments that systematically manipulate the communication design and subject pool.

For mixed motive games the picture seems clearer, although this may be due to fewer studies. Cooper et al. (1989) find that one-way communication results in a high degree of coordination in Battle of the Sexes. Averaged over several rounds of play, Cooper et al. (1989) report that one-way communication increases coordination from 48 percent without communication to 95 percent with one-way communication. With one round of two-way communication, coordination is 55 percent.¹⁷ For a comparison of this evidence with the prediction of the rational cheap talk model, see Costa-Gomes (2002).

To summarize, we believe that more experimental work is needed in order to test the theory laid out in this paper. Such a test should focus on players' initial responses to several different games, which would allow a clearer separation of types. Costa-Gomes and Crawford (2006) illustrates how this can be done. It would also be useful to directly test the assumption about T_0 players. Since T_0 players mainly exist in the minds of other players, we need data on players' beliefs. Such data can be generated not only through belief elicitation (Costa-Gomes and Weizsäcker 2007), but also by response time measurement (Rubinstein 2007), information search (Costa-Gomes et al. 2001 and Costa-Gomes and Crawford 2006) and through neuroimaging (Bhatt and Camerer 2005).

5 Concluding Remarks

The level- k model of bounded rationality captures many long-held intuitions both about the plausibility of Nash equilibrium play and about equilibrium selection. If players cannot com-

¹⁷It should be noted, however, that Cooper et al. (1989) allow the players to be silent and that 27 percent of the players in the two-way treatment, and 5 percent in the one-way treatment, choose to do so. We have not allowed silence in our analysis. It is of course possible to extend the message space to allow for silence, but we have chosen not to do so. Since players are assumed to have a slight preference for truthfulness, they might want to be silent when they don't know what action they are going to take in the action game (as T_1 under two-way communication in coordination games).

municate, the model provides a precise sense in which equilibrium is unlikely in the High Risk game, and it also correctly predicts play of the risk dominant equilibrium in Stag Hunt. Our analysis demonstrates that the level- k model also allows a number of sharp and non-trivial predictions concerning the role of communication in non-zero-sum games.

For pre-play communication in the class of symmetric 2×2 games, we are able to characterize precisely the outcomes in all games and for all type distributions. Arguably, our most remarkable result is the proof that communication can create reassurance in coordination games even if messages are highly unlikely to be self-signaling. When players are sufficiently sophisticated, the mere belief that some player type thinks that some player type thinks...etc...that a message is self-signaling suffices to uniquely select the efficient outcome in Stag Hunt with communication. When there are relatively unsophisticated (level-1) players in the population, we moreover find that two-way communication may yield higher expected payoff in Stag Hunt than does one-way communication. The latter result is typically reversed in mixed motive games: when players rank equilibria differently, average payoffs are usually higher under one-way communication.

While we show that communication also has beneficial effects in general two-player common interest games, not all results from our analysis of symmetric 2×2 games extend readily to other classes of games. In particular, we demonstrate by example that one-way communication sometimes hampers coordination, unless players think implausibly many steps.

Appendix 1: Characterization of Behavior

We here characterize behavior in all symmetric and generic 2×2 games using the level- k model. Consider the symmetric 2×2 game in Figure A1.

	H	L
H	u_{HH}, u_{HH}	u_{HL}, u_{LH}
L	u_{LH}, u_{HL}	u_{LL}, u_{LL}

Figure A1. Symmetric 2×2 game

We assume that this game is generic in the sense that none of the four different payoffs (u_{HH}, u_{HL}, u_{LH} and u_{LL}) are identical. Depending on the relations $u_{HH} \lesseqgtr u_{LH}$ and $u_{LL} \lesseqgtr u_{HL}$, we can divide the class of generic 2×2 games into three familiar types of games as shown in Figure A2.¹⁸

¹⁸The classification of symmetric games follows Weibull (1995) closely. To understand how this classification arises, note that if we were only interested in Nash equilibria of 2×2 games, we could have subtracted u_{LH} from both action H and L when the other player plays H and u_{HL} from both actions when the other player plays L . This would leave the equilibria of the game unchanged, whereas it affects the prediction for level- k models. The main reason is that in a level- k model, strategic uncertainty plays a role due to the randomization of level-0 players and we can therefore not use the sure-thing principle to transform the game. After the transformation,

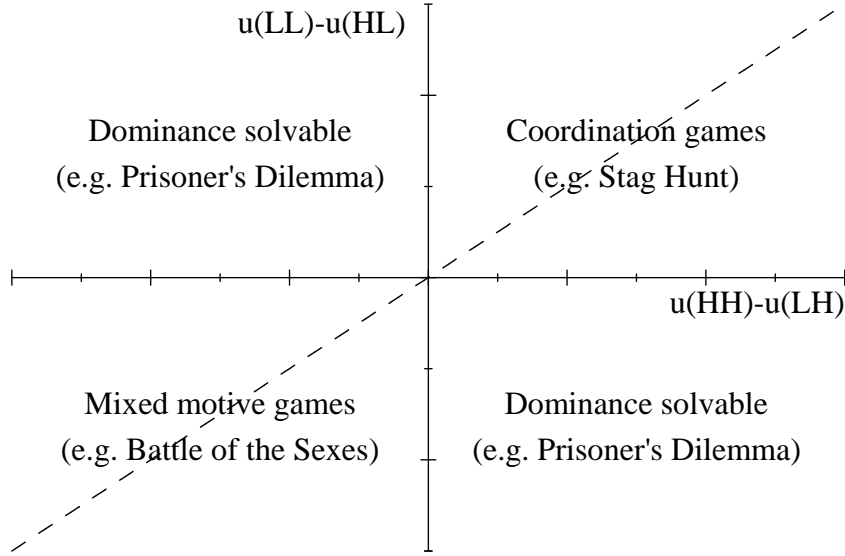


Figure A2. The four types of generic and symmetric 2×2 games

If we were only interested in Nash equilibria, there would be only one prediction for each of these games. For the level- k model, however, these games will be divided into subclasses with different predictions. The most important distinction is indicated by the dashed line in the figure. This condition corresponds to whether $u_{LL} - u_{HL} \leq u_{HH} - u_{LH}$, i.e., whether $u_{LH} + u_{LL} \leq u_{HH} + u_{HL}$. This means that action H is risk dominant above the dashed line in Figure A2, whereas action L is risk dominant below it. For tractability, we disregard the cases when neither action is risk dominant throughout the paper.

Dominance solvable games

Dominance solvable games are easiest to analyze, but also least interesting. In a dominance solvable game, players always have an incentive to play the dominant action, and neither one-way or two-way communication affect the actions players take.

We assume $u_{HL} > u_{LL}$ and $u_{HH} > u_{LH}$ so that H (igh) is the dominant action. The case when L is the dominant action is symmetric.

the game is the following.

	H	L
H	$u_{HH} - u_{LH}, u_{HH} - u_{LH}$	$0, 0$
L	$0, 0$	$u_{LL} - u_{HL}, u_{LL} - u_{HL}$

From this game it is clear why the class of symmetric games can be classified by two real numbers, $u_{HH} - u_{LH}$ and $u_{LL} - u_{HL}$.

Observation 1 *If players cannot communicate, T_{1+} plays the dominant action H . If players can communicate, then both one-way and two-way communication implies that T_{1+} sends h and plays H irrespective of any received messages.*

Proof. Since H is a dominant action, T_{1+} players play H irrespective of the believed behavior of the opponent. With the possibility to communicate, this also implies that there are no players that respond to messages, and T_{1+} players are therefore indifferent about sending h or l . (Sending l would have been beneficial if some players responded to messages and $u_{HL} > u_{HH}$ as in the Prisoner's Dilemma.) However, since players have a lexicographic preference for truthfulness, they send h . ■

For dominance solvable 2×2 games, communication plays no role. Except for some miscoordination due to T_0 playing the dominated action, all players play the dominant action. Since the proof only relies on the fact that each player has a strictly dominant strategy, the result extends to all normal form two-player games in which both players have a strictly dominant action.

Coordination games

Behavior in coordination games depends crucially on payoff and risk dominance. Since we restrict attention to generic games, one of the equilibria has to be payoff dominant. Let us without loss of generality assume that H (igh) is the payoff dominant equilibrium, i.e., $u_{HH} > u_{LL}$.

Observation 2 *(No communication) T_{1+} plays the risk dominant action.*

Proof. T_1 players believe that the opponent randomizes uniformly and therefore plays the risk dominant action. T_2 players best respond and play the same risk dominant action, and so on. ■

Absent communication, T_1 plays the best response to a uniformly randomizing T_0 opponent, which is the risk dominant action. Since this is a coordination game, more advanced players best respond by playing the same action.

Observation 3 *(One-way communication) If H is the risk dominant action, T_{1+} sends h and plays H as sender and responds to messages as receiver. If L is the risk dominant action, T_1 sends l and plays L as sender and responds to messages as receiver. T_{2+} sends h and plays H as sender and responds to messages as receiver.*

Proof. First consider the case when H is risk dominant. T_1 plays $\langle h, H, H, L \rangle$ (facing randomizing T_0 receivers and truthful T_0 senders). A T_2 sender believes that the receiver best-responds to the sent message and therefore sends h and plays H . A T_2 receiver believes that

the sender will send h and play H , but if T_2 receives message l , he believes it comes from a truthful T_0 sender. T_{2+} therefore plays $\langle h, H, H, L \rangle$.

Now consider the case when L is risk dominant. Then, T_1 plays $\langle l, L, H, L \rangle$. T_{2+} believes that the opponent responds to messages and that all messages are truthful and therefore play $\langle h, H, H, L \rangle$. ■

When risk and payoff-dominance coincide, one-way communication is sufficient to achieve coordination among T_{1+} players. When there is a conflict between risk and payoff dominance, there is still perfect coordination among T_{1+} players, but there is more play of the risk dominant equilibrium (since a T_1 sender plays the action corresponding to that equilibrium).

Observation 4 (*Two-way communication*) T_1 randomizes messages and responds to received messages, whereas T_{2+} sends h and plays H .

Proof. T_1 believes that the opponent is truthful and therefore best responds to the received message, while sending random messages (not knowing what action will be taken). T_2 believes that the opponent responds to messages and therefore sends and plays H irrespective of the message received (since T_1 sends a random message). T_3 therefore sends h and plays H . Receiving an unexpected L message, T_3 also plays H , believing the opponent to be T_1 . More advanced players reason in the same way and thus also play $\langle h, H, H \rangle$. ■

Mixed motive games

Two common examples of 2×2 mixed motive games are Chicken or Hawk-Dove and Battle of the Sexes. In order for the game to have mixed motive, we assume $u_{HL} > u_{LL}$ and $u_{LH} > u_{HH}$. Without loss of generality, we further assume that $u_{HL} > u_{LH}$ so that each player prefer the equilibrium where he is the one to play H (igh). If $u_{LL} = u_{HH} = 0$, then this game is the Battle of the Sexes, whereas it is a Chicken game if $u_{LL} > u_{HH}$. Battle of the Sexes is a non-generic game, but the results in this section hold also for the Battle of the Sexes.

Observation 5 (*No communication*) If H is the risk dominant action, then T_k plays H if k is odd and L if k is even. If L is the risk dominant action, then T_k plays L if k is odd and H if k is even.

Proof. T_1 plays the risk dominant action and T_k best-responds to the behavior of T_{k-1} , which generates the alternating behavior. ■

With no possibility to communicate, there is little players can do to coordinate on either of the asymmetric equilibria and behavior therefore alternates over thinking steps. One-way communication, on the other hand, provides a way to break the symmetry inherent in the game.

Observation 6 (*One-way communication*) If H is the risk dominant action, then T_{1+} sends h and plays H as sender and responds to messages as receiver. If L is the risk dominant action, then T_1 sends l and plays L as sender and responds to messages as receiver. T_{2+} sends h and plays H as sender and responds to messages as receiver.

Proof. First let H be the risk dominant action. A T_1 sender faces a randomizing receiver and therefore plays H and sends h . A T_1 receiver, on the other hand, responds to the sent message, believing it comes from a truthful T_0 opponent. T_{2+} can get the preferred equilibrium as sender and therefore sends h and plays H , while responding to messages as receiver. If instead L is the risk dominant action, a T_1 sender instead sends and plays L , but otherwise behavior is unchanged. ■

In general, senders play their preferred equilibrium and receivers yield and play their least preferred equilibrium. However, if the preferred equilibrium does not coincide with the risk dominant action, T_1 senders send and play their least preferred equilibrium.¹⁹

Observation 7 (*Two-way communication*) T_1 sends h and l with equal probabilities and responds to messages. The behavior of T_{2+} players cycles in thinking steps of six as follows: $\langle h, H, H \rangle, \langle l, L, L \rangle, \langle h, L, H \rangle, \langle h, H, H \rangle, \langle l, L, H \rangle, \langle h, L, H \rangle$.

Proof. T_1 believes that the opponent is truthful and therefore sends random messages, but responds to the message sent. T_2 believes that the opponent responds to messages and therefore plays $\langle h, H, H \rangle$. T_3 expects to receive a truthful h message, and thus sends l and plays L . If receiving an l message, T_3 believes it comes from a T_1 opponent and therefore plays L (believing the opponent will play H). T_4 expects to play H and therefore sends h . If receiving the message h , T_4 believes it comes from a T_2 opponent and therefore responds by playing L . T_5 thinks the opponent responds to messages and therefore plays H and sends h . Believe an l message comes from a T_2 opponent, T_5 subsequently plays H . T_6 expects to play L and therefore sends l , but plays H upon receiving an l message (believing it comes from a T_2 opponent). T_7 expects to play H and sends an h message, playing L if receiving an h message. T_8 sends h and plays H , playing H if he receives an l message, just like T_2 . T_9 plays $\langle l, L, L \rangle$ just like T_3 . Since the behavior of eight and nine-level players is just like two- and three-level players, and the rationale for T_{4+} did not depend on the behavior of T_0 or T_1 , behavior continues to cycle like this. ■

Note that the behavior of T_0, T_1, T_2 , and T_3 is identical to Crawford (2007). However, T_4 responds to received messages in our model, but always plays H in Crawford (2007). The difference stems from the fact that we assume that whenever T_4 receives the message h , the inference is that it comes from a T_2 player that will actually play H , whereas Crawford (2007) assumes that T_4 believes an h message is a mistake by a T_3 opponent who will play H anyway.²⁰

¹⁹The result when L is risk dominant is sensitive to the assumption that T_{1+} players have lexicographic preferences for truthfulness. Without that preference, level-1 senders would send random messages. Then, the behavior of more advanced players would alternate and entail many instances of miscoordination.

²⁰Also note that although our T_3 behaves as in Crawford (2007), the rationale for their behavior is slightly different. T_3 in our framework believes an l message comes from a T_1 opponent that sends random messages. Since T_3 sent the message l , the player believes that the opponent will play H and they therefore play L . In Crawford (2007), a T_3 player that receives the counterfactual message l believes that it was a mistake by the T_2 opponent and therefore plays L anyway.

Comparing one-way and two-way communication, it is clear that two-way communication will lead to several instances of miscoordination. However, as pointed out by Crawford (2007), the degree of coordination may still be higher than predicted by Farrell (1987) and Rabin (1994).

Finally, note the parallel to coordination games that risk-dominance only plays a role with one-way communication. The underlying reason is the strategic uncertainty resulting from randomizing T_0 receivers.

Appendix 2: Proofs

Proof of Proposition 1

From Observation 1 we know that communication has no effect in dominance solvable games. Similarly, for coordination games when H is risk dominant, Observation 2 and 3 show that communication has no effect. In coordination games when L is risk dominant, however, Observation 2 and 3 show that one-way communication results in either (L, L) or (H, H) , whereas no communication results in (L, L) . As long as there is a positive fraction of T_{2+} players, one-way communication therefore results in higher expected payoffs.

For mixed motive games, first suppose L is risk dominant. From Observation 6 we know that one-way communication always induces coordination when T_{1+} play, so the expected payoff for a player playing the game is $(u_{HL} + u_{LH})/2$. However, as noted in Observation 5, no communication results in miscoordination when two odd-level players meet as well as when two even-level players meet. Under the standard type distribution, a player's average payoff is

$$p_2^2 u_{HH} + p_2(1-p_2)u_{HL} + (1-p_2)p_2 u_{LH} + (1-p_2)^2 u_{LL}.$$

One-way communication results in higher expected payoff whenever

$$\left(\frac{1}{2} - p_2(1-p_2)\right)(u_{HL} + u_{LH}) > p_2^2 u_{HH} + (1-p_2)^2 u_{LL}.$$

A sufficient condition is that $u_{LL} < u_{HL}$ (we already know that $u_{HH} < u_{LH}$), but the necessary condition depends on p_2 . Now let H be the risk dominant outcome. The expected payoff for communicating players is unchanged, whereas the condition for one-way communication to result in higher expected payoff is

$$\left(\frac{1}{2} - p_2(1-p_2)\right)(u_{HL} + u_{LH}) > (1-p_2)^2 u_{HH} + p_2^2 u_{LL}.$$

Proof of Corollary 1

From the proof of Proposition 1 it follows directly that one-way communication only decreases average payoffs if one of the conditions hold with opposite inequality. To see why the corresponding game is a Chicken, suppose first that L is risk dominant. The first condition in Proposition 1 for one-sided communication to decrease expected payoffs is

$$\left(\frac{1}{2} - p_2(1 - p_2)\right)(u_{HL} + u_{LH}) < p_2^2 u_{HH} + (1 - p_2)^2 u_{LL}. \quad (1)$$

We know that $u_{HL} > u_{LL}$, $u_{LH} > u_{HH}$ and $u_{HL} > u_{LH}$. This implies that $u_{HH} < (u_{LH} + u_{HL})/2$. Suppose that $u_{LL} \leq (u_{LH} + u_{HL})/2$. Then the right hand side of (1) satisfies

$$\begin{aligned} p_2^2 u_{HH} + (1 - p_2)^2 u_{LL} &< p_2^2 \frac{1}{2}(u_{LH} + u_{HL}) + \frac{1}{2}(1 - p_2)^2 (u_{LH} + u_{HL}) \\ &= \left(\frac{1}{2} - p_2(1 - p_2)\right)(u_{LH} + u_{HL}). \end{aligned}$$

This implies that (1) cannot hold, and therefore the condition must fail unless $u_{LL} > \frac{1}{2}(u_{LH} + u_{HL})$. This implies that $u_{LL} > u_{HH}$, which implies that it is a Chicken. An analogous argument can be made when H is risk dominant.

Proof of Proposition 2

From Observation 1 we know that communication has no effect in dominance solvable games.

From Observation 2 and 3, we know that the outcomes of coordination games in which L is the risk dominant action. These are given in Table A1. Pairwise comparison of the cells in Table A1 reveals that one-way communication entails weakly more coordination.

G (no communication)			$\Gamma_I(G)$ (one-way communication)		
0	≥ 1		0R	1R	$\geq 2R$
0	Uniform	$\frac{1}{2}LL, \frac{1}{2}LH$	0S	Uniform	$\frac{1}{2}HH, \frac{1}{2}LL$ $\frac{1}{2}HH, \frac{1}{2}LL$
≥ 1	$\frac{1}{2}LL, \frac{1}{2}LH$	LL	1S	$\frac{1}{2}LL, \frac{1}{2}LH$	LL LL
			$\geq 2S$	$\frac{1}{2}HH, \frac{1}{2}HL$	HH HH

Table A1. Action profiles played in coordination games (L risk dominant)

If instead H is risk dominant, the outcomes are given in Table A2. The degree of coordination is again the same or higher with one-way communication than without communication.

G (no communication)			$\Gamma_I(G)$ (one-way communication)		
	0	≥ 1		0R	$\geq 1R$
0	Uniform	$\frac{1}{2}HH, \frac{1}{2}HL$	0S	Uniform	$\frac{1}{2}HH, \frac{1}{2}LL$
≥ 1	$\frac{1}{2}HH, \frac{1}{2}HL$	HH	$\geq 1S$	$\frac{1}{2}HH, \frac{1}{2}HL$	HH

Table A2. Action profiles played in coordination games (H risk dominant)

Now consider mixed motive games. Observations 5 and 6 yield the outcomes reported in Table A3 when L is risk dominant. Pairwise comparisons of cells reveal that the degree of coordination is higher with one-way communication.

G (no communication)				$\Gamma_I(G)$ (one-way communication)			
	0	Odd	Even		0R	1R	$\geq 2R$
0	Uniform	$\frac{1}{2}HL, \frac{1}{2}LL$	$\frac{1}{2}LH, \frac{1}{2}HH$	0S	Uniform	$\frac{1}{2}HL, \frac{1}{2}LH$	$\frac{1}{2}HL, \frac{1}{2}LH$
Odd	$\frac{1}{2}LH, \frac{1}{2}LL$	LL	LH	1S	$\frac{1}{2}LH, \frac{1}{2}LL$	LH	LH
Even	$\frac{1}{2}HL, \frac{1}{2}HH$	HL	HH	$\geq 2S$	$\frac{1}{2}HL, \frac{1}{2}HH$	HL	HL

Table A3. Action profiles played in mixed motive games (L risk dominant)

Finally, when H is risk dominant, the outcomes are given in Table A4. Again the degree of coordination is the same or higher for one-way communication for all combinations of types.

G (no communication)				$\Gamma_I(G)$ (one-way communication)		
	0	Odd	Even		0R	$\geq 1R$
0	Uniform	$\frac{1}{2}LH, \frac{1}{2}HH$	$\frac{1}{2}HL, \frac{1}{2}LL$	0S	Uniform	$\frac{1}{2}HL, \frac{1}{2}LH$
Odd	$\frac{1}{2}HL, \frac{1}{2}HH$	HH	HL	$\geq 1S$	$\frac{1}{2}HL, \frac{1}{2}HH$	HL
Even	$\frac{1}{2}LH, \frac{1}{2}LL$	LH	LL			

Table A4. Action profiles played in mixed motive games (H risk dominant)

Proof of Proposition 3

As Observation 1 shows, communication plays no role in dominance solvable games, so two-way communication cannot increase expected payoffs. In coordination games in which H is risk dominant, Observation 3 and 4 imply that $\Gamma_I(G)$ and $\Gamma_{II}(G)$ yield identical outcomes unless two T_1 players meet. In $\Gamma_I(G)$, players then coordinate on (H, H) , whereas there is miscoordination in $\Gamma_{II}(G)$. Thus $\Gamma_I(G)$ is weakly better than $\Gamma_{II}(G)$ in this case. When instead L is the risk dominant action, T_1 senders always play L . The average payoff associated with $\Gamma_I(G)$ is thus

$$p_1(1-p_1)u_{LL} + p_1(1-p_1)u_{HH} + (1-p_1)(1-p_1)u_{HH} + p_1^2u_{LL}.$$

The average payoff associated with $\Gamma_{II}(G)$ is

$$2p_1(1-p_1)u_{HH} + (1-p_1)(1-p_1)u_{HH} + \frac{1}{4}p_1^2(u_{LL} + u_{HH} + u_{LH} + u_{HL}).$$

Two-way communication thus yields higher payoff whenever

$$(4-3p_1)u_{HH} + p_1(u_{LH} + u_{HL}) > (4-p_1)u_{LL}.$$

Now consider mixed motive games. Observation 6 shows that for T_{1+} players, $\Gamma_I(G)$ entails perfect coordination, implying an average payoff of $(u_{LH} + u_{HL})/2$. As shown in Observation 7, matters are generally more complicated for $\Gamma_{II}(G)$ since behavior cycles over six thinking steps. Table A5 provides the resulting outcomes when confining attention to standard type distributions.

$\Gamma_{II}(G)$ (<i>two-way communication</i>)			
	1	2	3
1	Uniform	LH	HL
2	HL	HH	HL
3	LH	LH	LL

Table A5. Action profiles played in mixed motive games

We know that $(u_{LH} + u_{HL})/2 > u_{HH}$. However, if $u_{LL} > (u_{LH} + u_{HL})/2$ then two-way communication might be preferable. Two-way communication is preferable to one-way com-

munication whenever

$$\left(p_2 p_1 + p_1 p_3 + p_2 p_3 + \frac{1}{4} p_1^2\right) (u_{HL} + u_{LH}) + \left(p_3^2 + \frac{1}{4} p_1^2\right) u_{LL} + \left(p_2^2 + \frac{1}{4} p_1^2\right) u_{HH} > \frac{1}{2} (u_{LH} + u_{HL}).$$

Letting $p_2 = (1 - p_1 - p_3)$ we can rewrite this as

$$\frac{u_{LL} - u_{HH}}{u_{LH} + u_{HL} - 2u_{HH}} > 1 + \frac{2(p_1 - 1)(p_1 - 1 + 2p_3)}{p_1^2 + 4p_3^2}.$$

A necessary condition for this inequality to hold is that $u_{LL} > (u_{LH} + u_{HL})/2$. This follows from the fact that the minimum of the right hand side is $1/2$, whereas the left hand side can only be larger than $1/2$ if $u_{LL} > (u_{LH} + u_{HL})/2$.

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