Exercise 1 *Show that when markets are complete, the constraint*

$$C = Y + d \qquad d \in \underline{D}$$

can be rewritten as

$$C_0 + E_0 \left(\sum_{t=1}^{T} \Lambda_t C_t \right) = Y_0 + E_0 \left(\sum_{t=1}^{T} \Lambda_t Y_t \right).$$

(Hint: use a similar reasoning to the one presented when showing the corresponding result for the two-period case in Section 1.3 of the notes).

Exercise 2 *Prove Theorem 3.7. (Hint: use the result in Exercise 1 and apply the arguments of the proof of Theorem 1.10).*

Exercise 3 *The first-order conditions for the infinite horizon consumption-portfolio choice problem under standard assumptions on preferences are (in vector form)*

$$p_t u'(C_t^*) = \beta E_t \left[u'\left(C_{t+1}^*\right)(p_{t+1} + D_{t+1}) \right], \qquad t = 0, 1, \ldots \tag{1}$$

a. *Show that if the transversality condition*

$$\lim_{\tau \longrightarrow \infty} \beta^\tau u'(C_\tau)\, p_\tau = 0$$

holds, then (1) is equivalent to

$$p_t u'(C_t^*) = E_t \left[\sum_{\tau=t+1}^{\infty} \beta^\tau u'(C_\tau^*)\, D_\tau \right], \qquad t = 0, 1, \ldots$$

Exercise 4 *Consider an economy with complete markets and suppose that the representative agent has period utility of the form*

$$u(C) = \ln C.$$

There is only one firm which produces the single-good of the economy which is perishable (it can not be stored). The number of shares of this company is one and its owner is entitled to the company's total production every year $\overline{Y}_t$*. All other assets are in zero-net supply. The representative agent is endowed at time* 0 *with this share and* $\overline{Y}_0$ *(the production of the company at time* 0*) and no endowment at future dates.*

a. *Show that the price/dividend ratio of the only share of the economy must be constant in equilibrium. Find its expression. (Hint: assume that the corresponding transversality condition holds and write the first-order conditions in terms of all future dividends)*

b. *Find the expression of the ratio Λ_{t+1}/Λ_t for the SDF process Λ implied by the model in terms of the return of the unique asset in positive net-supply.*

Exercise 5 *Consider a dynamic economy where returns and consumption growth are conditionally joint lognormal and homoskedastic In the canonical case the Euler equations give that*

$$E_t\left[\beta\left(\frac{C^*_{t+1}}{C^*_t}\right)^{-\gamma}R^m_{t+1}\right]=1$$

and

$$E_t\left[\beta\left(\frac{C^*_{t+1}}{C^*_t}\right)^{-\gamma}R^f_{t+1}\right]=1$$

where R^m_{t+1} the return of the market portfolio (or the return of the only asset in positive net-supply in an economy like the one described in Exercise 4)..Show that

$$r^f_{t+1}=-\log\beta+\gamma E_t\left(\triangle c_{t+1}\right)-\frac{\gamma^2\sigma_c^2}{2}$$

and

$$E_t\left(r^m_{t+1}-r^f_{t+1}\right)+\frac{\sigma_m^2}{2}=\gamma\sigma_{mc},$$

where

$$\triangle c_{t+1}\equiv\ln\frac{C^*_{t+1}}{C^*_t}\qquad r^f_{t+1}\equiv\ln R^f_{t+1}\qquad r^m_{t+1}=\ln R_{t+1}$$

and

$$\sigma_c^2=var\left(\triangle c_{t+1}\right)\qquad\sigma_m^2=var\left(r^m_{t+1}\right)\qquad\sigma_{mc}=cov\left(r^m_{t+1},\triangle c_{t+1}\right).$$

Hint: recall that if

$$X\sim N\left(\mu,\sigma\right),$$

then

$$E\left(\exp X\right)=\exp\left(\mu+\frac{1}{2}\sigma^2\right).$$