North-South Trade with Increasing Product Variety

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Abstract: We present a model of one-way product cycles in international trade. Firms develop new product varieties in technologically advanced countries (the North), other firms copy these products in less developed countries (the South) and all shifts in production go from North to South. What distinguishes this paper from the earlier literature are the model's implications for economic growth and wage determination. Economic growth is characterized by weak scale effects, in contrast to the strong scale effects in the earlier literature. The model can also account for large North-South wage differences for plausible parameter values.

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1 Introduction

In a classic article, Vernon (1966) drew attention to the importance of product cycles in international trade. He pointed out that new products are typically introduced in technologically advanced countries (the North) and exported to less developed countries (the South). But as time passes, technology transfer to the lower-wage South takes place. These old products are then exported back to the North, reversing the initial pattern of international trade. Vernon's product cycles are readily observed in a wide variety of industries. For example, the production of refrigerators, microwave ovens and air conditioners used to be heavily concentrated in technologically advanced countries but today these products are manufactured mainly in China and other less developed countries.

Given the real world relevance of product cycles, economists have devoted considerable effort to developing dynamic models of North-South trade where new products are manufactured in the North and old products are manufactured in the South. Important contributions to this literature include Segerstrom, Anant and Dinopoulos (1990), Grossman and Helpman (1991a,b), Lai (1998), and Glass and Saggi (2002). In these models, northern firms engage in costly innovative R&D to develop new products and their behavior helps determine the rate of economic growth. There is also technology transfer to the South and the rate at which this transfer takes place helps to determine the size of the North-South wage ratio.

These early North-South trade models have a significant drawback: they all have the strong scale effect property, namely, that the growth rate of total factor productivity (TFP) depends positively on the scale of the economy. For example, these models imply that any increase in the size of the South permanently increases the TFP growth rate in the North. Since 1950, the South has increased dramatically in size, both due to population growth and to less developed countries like China opening up to international trade. But as Jones (1995a) points out, there has not been any upward trend in the TFP growth rates of advanced countries. Furthermore, these models imply that TFP growth is proportional to R&D employment, so if northern R&D employment doubles, then the northern TFP growth rate should also double. Since 1950, R&D employment has increased more than five-fold in advanced countries without generating any upward trend in TFP growth rates. The strong scale effect is clearly counterfactual.

In response to the Jones critique, a variety of closed-economy models have been developed that do not have the strong scale effect property, including Jones (1995b), Segerstrom (1998), and

Howitt (1999). Building on this literature, Dinopoulos and Segerstrom (2007), Parello (2007) and Sener (2006) have developed North-South trade models that do not have the strong scale effect. All of these models generate two-way product cycles: in each industry, production moves back and forth between the North and the South over time. Production moves from the North to the South when southern firms imitate northern products and production moves from the South to the North when northern firms develop higher quality products.

In this paper, we present a model of North-South trade that does not have the strong scale effect and generates one-way product cycles. New products are developed in the North and production moves from the North to the South when southern firms imitate northern products. But production never moves back to the North. All of the shifts in production are one-way: from North to South. For many products, once manufacturing leaves high-wage industrialized countries and moves to low-wage less developed countries, it never comes back. Our model is more in the spirit of Vernon's (1966) original discussion of product cycles.

The key difference between our model and the related literature are the assumptions about innovation. In Dinopoulos and Segerstrom (2007), Parello (2007) and Sener (2006), new products are higher quality versions of existing products and there is no change over time in the number of products consumed. In contrast, we study what happens when firms do innovative R&D to increase the number of products consumed, that is, to develop new product varieties. We build directly on the early work of Grossman and Helpman (1991a) and essentially present an improved version of their model.

The model has a unique steady-state equilibrium where the rate of innovation in the North, the rate of technology transfer to the South, and the North-South wage ratio are all constant over time. But unlike in the early literature on product cycles, economic growth is characterized by weak (instead of strong) scale effects. By weak scale effects, we mean that the level of TFP (not the growth rate) depends on the scale of the economy. Jones (2005) presents evidence that economic growth is in fact characterized by weak scale effects. Because we assume the same R&D technology as in Jones (1995b), our model is one of semi-endogenous growth.

In Grossman and Helpman (1991a), the central steady-state property is that an increase in the size of either region (the North or the South) increases the economic growth rate in both regions. This property does not hold in our semi-endogenous model where both regions are increasing in size over time and the common economic growth rate is constant. To illustrate the model's steady-state properties, we study the effects of three exogenous events tied to globalization: an increase in the relative size of the South in the world economy (e.g., countries like China joining the world trading system), an increase in the strength of intellectual property rights and a reduction in trade costs.

One challenge for North-South trade models is to explain the large wage differences between the North and the South. For example, when Sener (2006, Table 2b) solves his model numerically for plausible parameter values (the benchmark case), he finds that the northern wage rate is only 7 percent higher than the southern wage rate. Real world wage differences between developed and developing countries are obviously much larger. In this paper, we calibrate the model to match US-Mexico data. For plausible parameter values (the benchmark case), we obtain a northern wage rate that is 120 percent higher than the southern wage rate. Thus, the model can account for large wage differences between the North and the South.

The rest of the paper is organized as follows: In Section 2, we discuss properties of earlier North-South trade models to motivate the analysis that follows. In Section 3, we present the model and in Section 4, we show that it has a unique steady-state equilibrium. We solve analytically for the model's steady-state properties assuming costless trade in Section 5 and numerically assuming positive trade costs in Section 6. Finally, we offer some concluding remarks in Section 7.[1]

2 Motivation

In this section, we discuss why it is a challenge for North-South trade models to explain large wage differences between the North and the South. We use Sener (2006) to illustrate the issues and without fully specifying his model, we focus on key conditions that must be satisfied for the existence of a steady-state equilibrium with two-way product cycles.

In Sener (2006), the constant marginal cost of production in the North is the wage rate of northern labor w_N and the constant marginal cost of production in the South is the wage rate of southern labor w_S. In each industry, production jumps from the North to the South when a southern firm successfully copies a northern firm's product and production jumps from the South to the North when a northern firm successfully develops a higher quality version of a southern firm's product. Production only jumps from the North to the South following product imitation if the southern firm has a lower marginal cost of production than the northern firm that it copied. Thus, one condition that must hold in equilibrium is $w_N > w_S$. There is no incentive for southern firms to incur the positive costs of copying northern products if the southern wage rate is not lower than

[1]There is also an Appendix where calculations done to solve the model are spelled out in more detail and the benchmark parameter choices are fully explained. It can be downloaded at http://www2.hhs.se/personal/Segerstrom/.

the northern wage rate. Furthermore, production only jumps from the South to the North when a northern firm innovates if, when taking into account the size of the quality improvement $\lambda > 1$, the northern firm has a lower effective marginal cost than its southern rival. There is no incentive for northern firms to incur the positive costs of innovating if they cannot then compete against southern firms with lower wage costs. Thus, a second condition that must hold in equilibrium is $w_S > w_N/\lambda$. Putting these two conditions together, we obtain

$$\lambda > \frac{w_N}{w_S} > 1.$$

We can already see why North-South trade models have difficulty explaining large wage differences between the North and the South: the size of the quality improvement limits the North-South wage ratio that these models are capable of explaining. For example, if innovations are assumed to be 25 percent improvements in product quality ($\lambda = 1.25$), then these models can explain at most a northern wage rate that is 25 percent higher than the southern wage rate. Based on empirical evidence about markups of price over marginal cost, Sener (2006) chooses $\lambda = 1.25$ when he solves his model numerically and for plausible parameter values (the benchmark case in Table 2b), he obtains a northern wage rate that is only 7 percent higher than the southern wage rate.

Now consider what happens if we embellish Sener's model by allowing for positive trade costs and worker productivity differences between the North and the South. Suppose that there are trade costs separating the two regions that take the "iceberg" form: $\tau > 1$ units of a product must be produced and exported in order to have one unit arriving at its destination. Suppose also that one unit of northern labor produces $h > 1$ units of output whereas one unit of southern labor continues to produce one unit of output (northern workers have more human capital than southern workers). By assuming that northern workers are more productive than southern workers, there is more hope of explaining large wage differences between the North and the South.

Given the new assumptions, production only fully jumps from the North to the South following product imitation if the marginal cost of serving the northern market is greater for the northern firm than its southern imitator: $w_N/h > \tau w_S$. Furthermore, production only fully jumps from the South to the North following product innovation if the marginal cost of serving the southern market is lower for the northern innovator than the southern firm whose product has been improved upon:

$w_S > \tau w_N/(\lambda h)$. Putting these two conditions together, we obtain

$$\frac{\lambda h}{\tau} > \frac{w_N}{w_S} > \tau h.$$

Allowing for more productive northern workers (increasing h) does help in explaining a larger North-South wage ratio w_N/w_S. But as the trade cost parameter τ is increased, the largest North-South wage ratio that the model can account for shrinks and regardless of the value of h, the equilibrium involving two-way product cycles ceases to exist if $\lambda < \tau^2$. For example, if we continue to assume that innovations are 25 percent improvements in product quality ($\lambda = 1.25$), then the product-cycle equilibrium does not exist if trade costs exceed 12 percent ($\tau = 1.12$). Empirical estimates of trade costs far exceed this magnitude. Novy (2007) estimates that US trade costs with its major trading partners have declined on average from 83 percent in 1966 to 58 percent in 2002.[2] Even for the country Canada that the US trades the most with and has a free trade agreement with, trade costs are estimated to have exceeded 40 percent throughout the last four decades.

In this paper, we present a model of one-way product cycles, where all production shifts go from North to South. In our model, the equilibrium condition $\frac{w_N}{w_S} > \tau h$ has to hold for production to shift to the South following product imitation but production never shifts back to the North, so the equilibrium condition $\frac{\lambda h}{\tau} > \frac{w_N}{w_S}$ no longer applies. Thus, this model with one-way product cycles can potentially account for much larger wage differences between the North and the South.[3]

3 The Model

3.1 Overview

This is a model of trade between two regions: the North and the South. In both regions, labor is the only factor used to manufacture product varieties and to do R&D. Labor is perfectly mobile across activities within a region but cannot move across regions. Since labor markets are perfectly competitive, there is a single wage rate paid to all northern workers and a single wage rate paid to all southern workers. The two regions are distinguished by their R&D capabilities. Workers

[2] Trade cost estimates of similar magnitudes are reported in Anderson and van Wincoop (2003).

[3] We do not mean to deny the importance of quality improvements in the growth process. The model developed in this paper could be extended so that, in addition to variety-increasing R&D, firms also do R&D to increase the quality of their own products in each region. Such a model would still have one-way product cycles since the improvements in own product quality would not lead to production shifting between regions. Thus, the ideas developed in this paper have the potential to also explain large North-South wage differences in models with both increasing variety and increasing quality.

in the North are capable of conducting both innovative and imitative R&D whereas workers in the South can only conduct imitative R&D. We focus on the steady-state properties of the model where all innovative activity takes place in the high-wage North and all imitative activity takes place in the low-wage South. Northern firms engage in innovative R&D to extend the number of product varieties available to world consumers while southern firms engage in imitative R&D to shift production of existing varieties to the South. The model differs from Grossman and Helpman (1991b) by allowing for trade costs, population growth, human capital differences across regions, and by modeling R&D as in Jones (1995b).

3.2 Households

In both regions, there is a fixed measure of households that provide labor services in exchange for wage payments. Each individual member of a household lives forever and is endowed with one unit of labor, which is inelastically supplied. The size of each household, measured by the number of its members, grows exponentially at a fixed rate $g_L > 0$, the population growth rate. Let $L_{it} = L_{i0}e^{g_L t}$ denote the supply of labor in region i at time t ($i = N, S$), and let $L_t = L_{Nt} + L_{St}$ denote the world supply of labor. In addition to wage income, households also receive asset income from their ownership of firms. Although many of the model's properties do not depend on who owns the firms, to be concrete, we assume that firms in region i are owned by households in region i ($i = N, S$) and solve for the behavior of the representative consumer in each region. Let c_{it} denote the representative consumer's expenditure in region i at time t.

Households in both regions share identical preferences. Each household is modeled as a dynastic family that maximizes discounted lifetime utility

$$U_i = \int_0^\infty e^{-(\rho - g_L)t} \ln\left[u_{it}\right] dt \tag{1}$$

where $\rho > g_L$ is the subjective discount rate and u_{it} is the static utility of the representative household member in region i at time t. The static CES utility function is given by

$$u_{it} = \left[\int_0^{n_t} x_{it}\left(\omega\right)^\alpha d\omega\right]^{\frac{1}{\alpha}} \qquad 0 < \alpha < 1 \tag{2}$$

where $x_{it}(\omega)$ is the quantity consumed of product variety ω by the representative consumer in region i at time t, n_{Nt} is the number of varieties produced in the North, n_{St} is the number of varieties produced in the South, and $n_t = n_{Nt} + n_{St}$ is the number of varieties available on the world market. We assume that varieties are gross substitutes. Then with α measuring the degree of product differentiation, the elasticity of substitution between product varieties is $\sigma = \frac{1}{1-\alpha} > 1$.

Solving the static consumer optimization problem yields the familiar demand function

$$x_{it}(\omega) = \frac{p_{it}(\omega)^{-\sigma} c_{it}}{P_{it}^{1-\sigma}} \tag{3}$$

where $p_{it}(\omega)$ is the price of variety ω and $P_{it} \equiv \left[\int_0^{n_t} p_{it}(\omega)^{1-\sigma} d\omega\right]^{1/(1-\sigma)}$ is an index of consumer prices. Maximizing (1) subject to (2) where (3) has been used to substitute for $x_{it}(\omega)$ yields the Euler-condition

$$\frac{\dot{c}_{it}}{c_{it}} = r_t - \rho \tag{4}$$

implying that the representative consumer's expenditure in each region grows over time only if the market interest rate r_t exceeds the subjective discount rate ρ.

We solve the model for a steady-state equilibrium where both wage rates and consumer expenditure levels are constant over time. Let w_i denote the wage rate in region i and dropping the time subscript, let c_i henceforth denote the expenditure level of the representative consumer in region i. In steady-state equilibrium, (4) implies that the market interest rate must also be constant over time and given by $r_t = \rho$. We treat the southern wage rate as the numeraire price ($w_S = 1$), so all prices are measured relative to the price of southern labor.

3.3 Product Markets

The firms producing different product varieties compete in prices and maximize profits. The production of output is characterized by constant returns to scale. For each southern firm that knows how to produce a product variety, one unit of labor produces one unit of output. For each northern firm that knows how to produce a product variety, one unit of labor produces $h > 1$ units of output. The parameter h is intended to capture differences in worker productivity due to northern workers having higher levels of human capital than southern workers ("h" for human capital). Thus, each firm in the North has a constant marginal cost of production equal to w_N/h and each firm in the South has a constant marginal cost of production equal to w_S. There are also trade costs separating the two regions that take the "iceberg" form: $\tau \geq 1$ units of a variety must be produced and exported in order to have one unit arriving at its destination.[4]

Production only completely shifts from the North to the South following product imitation if the effective marginal cost of serving the northern market is greater for the northern firm than its southern imitator: $w_N/h > \tau w_S$. We solve the model for a steady-state equilibrium where this inequality holds, that is, the North-South wage ratio w_N/w_S satisfies $w_N/w_S > h\tau$ (or $w_N > h\tau$). In equilibrium, northern workers earn a higher wage than southern workers.

At each point in time, a firm can choose to shut down its manufacturing facilities and once it has done so, this decision can only be reversed by incurring a positive entry cost. Furthermore, each firm that fails to attract any consumers (has zero sales) incurs a positive cost of maintaining its unused manufacturing facilities, in addition to the constant marginal cost of production mentioned above. Thus firms that are not able to attract any consumers (because of relatively high labor costs) choose to shut down their manufacturing facilities in equilibrium and do not play any role in determining market prices, as in Segerstrom (2007, p.253).

In the presence of trade costs, northern consumers face different prices than southern consumers and we need to take this into account. Using (3), the northern consumer's demand for a domestically produced variety is $x_{Nt} = (p_{Nt})^{-\sigma} c_N/(P_{Nt})^{1-\sigma}$ and the northern consumer's demand for an imported good (exported by the South) is $x^*_{St} = (p^*_{St})^{-\sigma} c_N/(P_{Nt})^{1-\sigma}$ where a star denotes exports and subscripts denote production location. In these expressions, the northern price index P_{Nt} satisfies $(P_{Nt})^{1-\sigma} = \int_{n_{Nt}} p_{Nt}(\omega)^{1-\sigma} d\omega + \int_{n_{St}} p^*_{St}(\omega)^{1-\sigma} d\omega$, where n_{Nt} is the set of industries with northern production, and n_{St} is the set of industries with southern production. Similarly, the southern consumer's demand for a domestically produced variety is $x_{St} = (p_{St})^{-\sigma} c_S/(P_{St})^{1-\sigma}$ and the southern consumer's demand for an imported variety (exported by the North) is $x^*_{Nt} = (p^*_{Nt})^{-\sigma} c_S/(P_{St})^{1-\sigma}$ where the southern price index P_{St} satisfies $(P_{St})^{1-\sigma} = \int_{n_{Nt}} p^*_{Nt}(\omega)^{1-\sigma} d\omega + \int_{n_{St}} p_{St}(\omega)^{1-\sigma} d\omega$.

Consider now the profit-maximization decision of a northern firm that produces variety ω at time t. Omitting the arguments of functions, export profits are given by $\pi^*_N = (p^*_N - w_N \tau h^{-1}) x^*_N L_S$. The firm supplies $x^*_N L_S$ units to southern consumers but has to produce $\tau x^*_N L_S$ units and pays its workers w_N/h for each unit produced. Maximizing π^*_N with respect to p^*_N yields the profit-maximizing export price $p^*_N = \frac{\tau w_N}{\alpha h}$, which is the standard monopoly markup of price over marginal cost. Domestic profits are given by $\pi^d_N = (p_N - w_N h^{-1}) x_N L_N$. Maximizing π^d_N with respect to

[4] All the theorems derived in this paper continue to hold if iceberg trade costs are replaced by tariffs. If we let τ denote one plus the common tariff rate, then the only change in the equations of section 3 is that the τ term disappears in the definitions of $\bar{x}_{Nt}$ and $\bar{x}_{St}$. In section 4, the northern and southern steady-state equations remain unaffected but the steady-state wage equation does change.

p_N yields the profit-maximizing domestic price $p_N = \frac{w_N}{\alpha h}$. The prices of domestically produced varieties are lower because exporters need to compensate for the extra trade costs involved in their sales. Taking into account both domestic and export profits, the total profit flow $\pi_N = \pi_N^d + \pi_N^*$ of a northern firm is

$$\pi_{Nt} = \frac{w_N \bar{x}_{Nt} L_t}{(\sigma - 1)h} \tag{5}$$

where $\bar{x}_{Nt} \equiv (x_{Nt}L_{Nt} + \tau x_{Nt}^* L_{St})/L_t$ is the per capita world demand for a northern variety.

To calculate the profits of a southern firm, we first consider the case where the North-South wage ratio is large ($w_N/h > \tau w_S/\alpha$). Then the southern firm can charge the unconstrained monopoly price and does not face effective competition from a northern firm producing the same product variety. For such a southern firm, export profits are given by $\pi_S^* = (p_S^* - \tau w_S)x_S^* L_N$. The firm supplies $x_S^* L_N$ units to northern consumers but has to produce $\tau x_S^* L_N$ units and pays its workers the southern wage rate w_S for each unit produced. Maximizing π_S^* with respect to p_S^* yields the profit-maximizing export price $p_S^* = \tau w_S/\alpha$. Domestic profits are given by $\pi_S^d = (p_S - w_S)x_S L_S$. Maximizing π_S^d with respect to p_S yields the profit-maximizing domestic price $p_S = w_S/\alpha$. Taking into account both domestic and export profits, the total profit flow $\pi_S = \pi_S^d + \pi_S^*$ of this southern firm is

$$\pi_{St} = \frac{w_S \bar{x}_{St} L_t}{\sigma - 1} \tag{6}$$

where $\bar{x}_{St} \equiv (x_{St}L_{St} + \tau x_{St}^* L_{Nt})/L_t$ is the per capita world demand for the southern variety.

In the case where the North-South wage ratio is small ($w_N/h \leq \tau w_S/\alpha$), we need to take into account possible competition from a northern firm producing the same product variety. Focusing on the export market (similar calculations apply for the domestic market), it is profit-maximizing for the southern firm to charge the limit price w_N/h until the northern firm shuts down its production facilities and then charge the unconstrained monopoly price $\tau w_S/\alpha$. Furthermore, it is profit-maximizing for the northern firm to shut down immediately since marginal cost pricing yields zero sales (assuming that indifferent consumers buy from the southern firm). Along the equilibrium path, northern firms immediately shut down when southern firms imitate their products and then southern firms charge unconstrained monopoly prices. Thus equation (6) also holds when the North-South wage ratio is small.

3.4 Innovation and Imitation

To innovate and develop a new product variety, a representative northern firm i must devote $a_N/(K_{Nt})^\theta$ units of labor to innovative R&D, where a_N is an innovative R&D productivity parameter, K_{Nt} is the disembodied stock of knowledge relevant for northern firms and $\theta > 0$ is an intertemporal knowledge spillover parameter.[5] The disembodied stock of knowledge K_{Nt} grows over time and is available to all northern firms. We assume that it is proportional to the total number of varieties that have been developed in the past and choose units so that $K_{Nt} \equiv n_t$. The restriction $\theta > 0$ implies that R&D labor becomes more productive as time passes and a northern firm needs to devote less labor to develop a new variety as the stock of knowledge increases. Grossman and Helpman (1991a) assume that intertemporal knowledge spillovers are quite strong and set $\theta = 1$. We will instead follow Jones (1995b) by assuming that intertemporal knowledge spillovers are weaker and satisfy $\theta < 1$. This later assumption is the key to ruling out strong scale effects.

Given the innovative R&D technology, the rate at which northern firm i discovers new products is $\dot{n}_{it} = l_{Rit}/[a_N/(K_{Nt})^\theta] = n_t^\theta l_{Rit}/a_N$ where $\dot{n}_{it}$ is the time derivative of n_{it} and l_{Rit} is the labor used for innovative activities by firm i ("R" for R&D). Summing over individual northern firms, the aggregate rate at which the North innovates is

$$\dot{n}_t = \frac{n_t^\theta L_{Rt}}{a_N} \tag{7}$$

where $L_{Rt} = \sum_i l_{Rit}$ is the total amount of northern labor employed in innovative activities.

Similarly, to learn how to produce a randomly chosen northern variety, a southern firm j must devote $a_S/(K_{St})^\theta$ units of labor to imitative R&D, where a_S is an imitative R&D productivity parameter, $K_{St} \equiv n_{St} + kn_{Nt}$ is the disembodied stock of knowledge relevant for southern firms and $k \in [0, 1]$ is a spillover parameter. We interpret a_S as measuring the strength of intellectual property rights (IPR) protection. A higher value of a_S then means that there is stronger IPR protection and it is harder for a southern firm to successfully copy a northern firm's product. As is the case with innovating, we assume that there are intertemporal knowledge spillovers associated with imitating. As the stock of knowledge K_{St} increases over time, copying northern products becomes easier given $\theta > 0$. $k = 0$ corresponds to no international knowledge spillovers ($K_{St} = n_{St}$) and $k = 1$ corresponds to perfect spillovers ($K_{St} = n_{St} + n_{Nt} = n_t$). Grossman and Helpman (1991a)

[5] Jones (1995b) allows for both cases $\theta > 0$ and $\theta < 0$, but we restrict attention to the first case since this simplifies some later proofs and in the section on numerical results, the first case turns out to be the empirically relevant one.

assume no spillovers ($k = 0$) in their main text but they also discuss in footnotes the implications of positive spillovers ($k \in (0, 1]$). We allow for both possibilities.

Given the imitative R&D technology, the rate at which southern firm j copies northern products is $\dot{n}_{Sjt} = l_{Cjt}/[a_S/(K_{St})^\theta] = (K_{St})^\theta l_{Cjt}/a_S$ where l_{Cjt} is the labor used for imitative activities by firm j ("C" for copying). Summing over individual southern firms, the aggregate rate at which the South imitates is

$$\dot{n}_{St} = \frac{(K_{St})^\theta L_{Ct}}{a_S} \tag{8}$$

where $L_{Ct} \equiv \sum_j l_{Cjt}$ is the total amount of southern labor employed in imitative activities.

3.5 R&D Incentives

A representative northern firm producing a variety that has not yet been imitated earns the profit flow π_{Nt} during the time interval dt. Northern firms face an ongoing risk of imitation and with random selection from a uniform distribution, any given northern firm will lose its monopoly position during such a time interval with probability $\dot{n}_{St}dt/n_{Nt}$. In the event that this occurs, owners of the firm will suffer a total capital loss of V_{Nt}, the stock-market value of the firm. If the variety is not imitated, firm owners instead receive the capital gain $\dot{V}_{Nt}$ during the time interval dt.

We assume that there is a stock market that channels consumer savings to northern and southern firms that engage in R&D and helps households to diversify the risk of holding stocks issued by these firms. Since there is no aggregate risk in the economy, it is possible for households to earn a safe return by holding the market portfolio in each region. Hence, ruling out any arbitrage opportunities implies that the total return on equity claims must equal the opportunity cost of invested capital, which is given by the risk-free market interest rate ρ. The northern no-arbitrage condition then becomes $\rho V_{Nt}dt = \pi_{Nt}dt - [\dot{n}_{St}/n_{Nt}]V_{Nt}dt + \dot{V}_{Nt}dt$. The return from holding the stock of a northern firm equals the return from an equal-sized investment in a riskless bond.

If innovative activity leads to pure economic profits, then entry of entrepreneurs generates excess demand for northern labor. Thus, free entry into innovative activities implies that the cost of innovating must be exactly balanced by the expected discounted profits from innovating. With innovative R&D being subsidized at the rate s_R, this yields

$$V_{Nt} = \frac{w_N a_N}{n_t^\theta}(1 - s_R). \tag{9}$$

Dividing the northern no-arbitrage condition by $V_{Nt}dt$ on both sides, defining the rate of imitation as $\mu \equiv \frac{\dot{n}_{St}}{n_{Nt}}$ and using (9), the northern no-arbitrage condition can be written as

$$\frac{\pi_{Nt}}{\rho + \mu - \frac{\dot{V}_{Nt}}{V_{Nt}}} = \frac{w_N a_N}{n_t^{\theta}}(1 - s_R). \tag{10}$$

Turning to the South, once a southern firm has successfully imitated a product variety produced in the North, production shifts to the South and the southern firm earns monopoly profits from then on. During a time interval dt, owners of the southern firm earn the profit flow $\pi_{St}dt$ and also realize the capital gain $\dot{V}_{St}dt$, where V_{St} is the expected discounted profits or market value of the southern firm. With no arbitrage opportunities, the total return on equity claims must equal the risk-free market interest rate, which implies that $\rho V_{St}dt = \pi_{St}dt + \dot{V}_{St}dt$. If imitative activity leads to pure economic profits, then entry of entrepreneurs generates excess demand for southern labor. Thus, free entry into imitative activities implies that the cost of imitating must be exactly balanced by the expected discounted profits from imitating. With imitative R&D being subsidized at the rate s_C, this yields

$$V_{St} = \frac{w_S a_S}{(K_{St})^{\theta}}(1 - s_C). \tag{11}$$

Dividing the southern no-arbitrage condition by $V_{St}dt$ on both sides and substituting for V_{St} using (11), the southern no-arbitrage condition can be written as

$$\frac{\pi_{St}}{\rho - \frac{\dot{V}_{St}}{V_{St}}} = \frac{w_S a_S}{(K_{St})^{\theta}}(1 - s_C). \tag{12}$$

3.6 Labor Markets

Labor markets are perfectly competitive and wages adjust instantaneously to equate labor demand and labor supply. In the North, labor is employed in production and innovative R&D. Each innovation requires a_N/n_t^{θ} units of labor, so the total employment of labor in innovative R&D is $a_N\dot{n}_t/n_t^{\theta}$. A northern firm uses $\bar{x}_{Nt}L_t/h$ units of labor to produce for both the domestic and export markets, and there are n_{Nt} varieties produced by northern firms, so the total employment of labor in northern production is $\bar{x}_{Nt}L_t n_{Nt}/h$. As L_{Nt} denotes the supply of labor in the North, full employment requires that

$$L_{Nt} = \frac{a_N}{n_t^{\theta}}\dot{n}_t + \frac{X_{Nt}L_t}{h} \tag{13}$$

where $X_{Nt} \equiv \bar{x}_{Nt}n_{Nt}$ is the per capita world demand for all northern varieties.

In the South, labor is employed in production and imitative R&D. Each imitation requires

$a_S/(K_{St})^\theta$ units of labor, so the total employment of labor in imitative R&D is $a_S\dot{n}_{St}/(K_{St})^\theta$. A southern firm uses $\bar{x}_{St}L_t$ units of labor to produce for both the domestic and export markets, and there are n_{St} varieties produced by southern firms, so the total employment of labor in southern production is $\bar{x}_{St}L_t n_{St}$. As L_{St} denotes the supply of labor in the South, full employment requires that

$$L_{St} = \frac{a_S}{(K_{St})^\theta}\dot{n}_{St} + X_{St}L_t \tag{14}$$

where $X_{St} \equiv \bar{x}_{St}n_{St}$ is the per capita world demand for all southern varieties.

This completes the description of the model.

4 Solving the model

In this section, we solve the model for a unique steady-state (or balanced growth) equilibrium where all endogenous variables grow over time at constant (not necessarily identical) rates. As we will show, solving the model for a steady-state equilibrium reduces to solving a simple system of two equations in two unknowns, where the two unknowns are the imitation rate μ and relative R&D difficulty δ_N (which is yet to be defined).

In any steady-state equilibrium, the share of labor employed in R&D activities must be constant over time (in both the North and the South). Given that the supply of labor in each of the two regions grows at the population growth rate g_L, northern R&D employment L_{Rt} and southern R&D employment L_{Ct} must grow at this rate as well. If we denote the steady-state growth rate of the number of varieties by $g \equiv \dot{n}_t/n_t$, then dividing both sides of (7) by n_t yields $g = \frac{n_t^{\theta-1}L_{Rt}}{a_N}$. It follows that g can only be constant over time if

$$g = \frac{g_L}{1-\theta}. \tag{15}$$

Thus, the steady-state rate of innovation g is pinned down by parameter values and is proportional to the population growth rate g_L. As in Jones (1995b), the parameter restriction $\theta < 1$ is needed to guarantee that the steady-state rate of innovation is positive and finite.

Equation (15) has two important implications, given that the innovation rate g is proportional to the economic growth rate (as we will show later in the paper). First, equation (15) implies that public policy changes like stronger intellectual property rights protection (an increase in a_S) or trade liberalization (a decrease in τ) have no effect on the steady-state rate of innovation g

and hence the steady-state rate of economic growth. In this model, growth is "semi-endogenous". We view this as a virtue of the model because both total factor productivity and per capita GDP growth rates have been remarkably stable over time in spite of many public policy changes that one might think would be growth-promoting. For example, plotting data on per capita GDP (in logs) for the US from 1880 to 1987, Jones (1995a) shows that a simple linear trend fits the data extremely well. This data leads us to be skeptical about models where public policy changes have large long-run growth effects. Second, equation (15) implies that the level of per capita income in the long run is an increasing function of the size of the economy. Jones (2005) has a lengthy discussion of this "weak scale effect" property and cites Alcala and Ciccone (2004) as providing the best empirical support. Controlling for both trade and institutional quality, Alcala and Ciccone (2004) find that a 10 percent increase in the size of the workforce in the long run is associated with 2.5 percent higher GDP per worker.[6]

Returning to solving the model, we now derive some steady-state equilibrium implications of the condition $n_{Nt} + n_{St} = n_t$. First, the number of varieties produced in each region must grow at the same rate as the total number of existing varieties: $g \equiv \frac{\dot{n}_t}{n_t} = \frac{\dot{n}_{Nt}}{n_{Nt}} = \frac{\dot{n}_{St}}{n_{St}}$. Second, the variety shares produced in the two regions $\gamma_N \equiv \frac{n_{Nt}}{n_t}$ and $\gamma_S \equiv \frac{n_{St}}{n_t}$ are necessarily constant over time and satisfy $\gamma_N + \gamma_S = 1$. Third, γ_N and γ_S can be determined by noting that $\frac{\dot{n}_t}{n_t} = \frac{\dot{n}_{Nt}}{n_{Nt}}\frac{n_{Nt}}{n_t} + \frac{\dot{n}_{St}}{n_{Nt}}\frac{n_{Nt}}{n_t}$ or $g = g\gamma_N + \mu\gamma_N$. Solving for the northern share of variety production and the corresponding southern share of variety production yields

$$\gamma_N = \frac{g}{g+\mu} \qquad \text{and} \qquad \gamma_S = \frac{\mu}{g+\mu}. \tag{16}$$

An increase in the rate of imitation μ naturally results in a higher share of varieties produced in the South and a lower share of varieties produced in the North.

We are now in a position to define relative R&D difficulty. Let the term $n_t^{-\theta}$ in (10) be a measure of (absolute) R&D difficulty. It decreases over time since $0 < \theta < 1$. Let L_{Nt}/n_t as a measure of the size of the northern market that firms sell to. Population growth increases the size of the

[6]In spite of the arguments in Jones (2005), the issue of whether economic growth is semi-endogenous or not remains controversial. For example, Ha and Howitt (2007) present evidence that "fully-endogenous" growth models (where public policy choices affect the steady-state rate of economic growth) have better empirical support than semi-endogenous growth models. We think that their analysis has an important limitation. Ha and Howitt implicitly assume that convergence to steady-state is fast, so that with 50 years of data, they can just focus on the steady-state implications of growth models. This assumption is called into question in Steger (2003), who calibrates a semi-endogenous growth model using US data and finds that convergence to steady-state is slow: it takes almost 40 years to go half the distance to the steady-state. With such slow convergence, we think that future tests of semi-endogenous growth theory should take into account the transition path implications of the theory.

market but variety growth has the opposite effect because firms have to share consumer demand with more competing firms.[7] By taking the ratio of R&D difficulty and the northern market size term L_{Nt}/n_t, we obtain a measure of relative R&D difficulty (or R&D difficulty relative to the size of the market):

$$\delta_N \equiv \frac{n_t^{-\theta}}{\frac{L_{Nt}}{n_t}} = \frac{n_t^{1-\theta}}{L_{Nt}}. \tag{17}$$

Log-differentiating (17) using (15), it is easily verified that δ_N is constant over time in any steady-state equilibrium. In any steady-state equilibrium with decreasing R&D difficulty, each firm faces a shrinking market size since population growth is exceeded by variety growth (L_{Nt}/n_t decreases over time).[8]

Using δ_N, we can derive a steady-state northern innovation condition. Differentiating (9) yields the northern capital gains rate $\dot{V}_{Nt}/V_{Nt} = -\theta g$ and the northern full employment condition (13) implies that $X_N \equiv \bar{x}_{Nt} n_{Nt}$ must be constant over time in any steady-state equilibrium. Substituting these equations into (10) using (5) and dividing both sides by the market size term L_{Nt}/n_t yields the northern innovation condition

$$\frac{\frac{X_N L_0}{(\sigma-1)h\gamma_N L_{N0}}}{\rho + \mu + \theta g} = a_N(1 - s_R)\delta_N. \tag{18}$$

The left-hand-side is the market size-adjusted benefit from innovating and the right-hand-side is the market size-adjusted cost of innovating. In steady-state calculations, we need to adjust for market size because market size changes over time. The market size-adjusted benefit from innovating is higher when the average world consumer buys more of each northern variety ($X_N/\gamma_N \uparrow$), there are more consumers in the world to sell to ($L_0 \uparrow$), future profits are less heavily discounted ($\rho \downarrow$), northern firms are less threatened by southern imitation ($\mu \downarrow$) and northern firms experiences larger capital gains over time ($\theta g \downarrow$). The market size-adjusted cost of innovating is higher when northern researchers are less productive ($a_N \uparrow$), northern R&D is subsidized less ($s_R \downarrow$) and innovating is relatively more difficult ($\delta_N \uparrow$).

Similarly, we can derive a steady-state southern imitation condition. Since $K_{St} \equiv n_{St} + kn_{Nt} = (\gamma_S + k\gamma_N)n_t$, and both variety shares γ_S and γ_N are constant over time in any steady-state

[7] As shown in the Appendix, π_{Nt} and π_{St} are both proportional to L_{Nt}/n_t and only change over time based on how L_{Nt}/n_t changes.

[8] We define relative R&D difficulty using the northern labor force (instead of the world labor force) in order to facilitate the comparative steady-state analysis in the next section. There, we study the long-run effects of an initial increase in the size of the South and because this increase does not cause δ_N to jump, δ_N can play the role of a state variable in the analysis.

equilibrium, $\dot{K}_{St}/K_{St} = g$. Differentiating (11) yields the southern capital gains rate $\dot{V}_{St}/V_{St} = -\theta g$ and the southern full employment condition (14) implies that $X_S \equiv \bar{x}_{St} n_{St}$ must be constant over time in any steady-state equilibrium. Substituting these equations into (12) using (6) and dividing both sides by the market size term L_{Nt}/n_t yields the southern imitation condition

$$\frac{\frac{X_S L_0}{(\sigma-1)\gamma_S L_{N0}}}{\rho + \theta g} = \frac{a_S(1-s_C)\delta_N}{(\gamma_S + k\gamma_N)^\theta}. \tag{19}$$

The left-hand-side is the market size-adjusted benefit from imitating and the right-hand-side is the market size-adjusted cost of imitating. The market size-adjusted benefit from imitating is higher when the average world consumer buys more of each southern variety ($X_S/\gamma_S \uparrow$), there are more consumers in the world to sell to ($L_0 \uparrow$), future profits are less heavily discounted ($\rho \downarrow$), and southern firms experiences larger capital gains over time ($\theta g \downarrow$). The market size-adjusted cost of imitating is higher when southern researchers are less productive ($a_N \uparrow$), southern R&D is subsidized less ($s_C \downarrow$) and imitating is relatively more difficult ($\delta_N/(\gamma_S + k\gamma_N)^\theta \uparrow$).

Using (17) and evaluating at time $t = 0$, the northern labor condition (13) can be rewritten as $L_{N0} = a_N \delta_N L_{N0} g + X_N L_0/h$. Substituting for $X_N L_0/h$ using (18), substituting for γ_N using (16), and dividing both sides by L_{N0} yields the *northern steady-state condition*

$$1 = a_N \delta_N g \left[1 + \left(\frac{\rho + \mu + \theta g}{g + \mu}\right)(\sigma - 1)(1 - s_R)\right]. \tag{20}$$

Equation (20) is a northern full employment condition that takes into account the implications of profit-maximizing R&D behavior by northern firms. A higher imitation rate μ decreases the right-hand-side, so relative R&D difficulty δ_N must increase to restore equality in (20), given that g is pinned down by (15).[9] It follows that the northern steady-state condition is upward-sloping in (δ_N, μ) space: any increase in the rate of imitation μ (which reduces northern production employment) must be matched by an increase in relative R&D difficulty δ_N (which raises northern R&D employment).

Similarly, using (17) and evaluating at time $t = 0$, the southern labor condition (14) can be rewritten as $L_{S0} = a_S \delta_N L_{N0} g \gamma_S (\gamma_S + k\gamma_N)^{-\theta} + X_S L_0$. Substituting for $X_S L_0$ using (19), substituting for γ_S using (16), substituting $\gamma_S + k\gamma_N = \frac{\mu + kg}{g+\mu}$ and then dividing both sides by L_{S0}

[9]This can be seen by using $g = \frac{g_L}{1-\theta}$ and noting that $\frac{\partial}{\partial \mu}\left[\frac{\rho+\mu+\theta g}{g+\mu}\right] = \frac{(g_L - \rho)}{(g+\mu)^2} < 0$.

yields the *southern steady-state condition*

$$1 = a_S \delta_N \frac{L_{N0}}{L_{S0}} \mu (g+\mu)^{\theta-1} (\mu+kg)^{-\theta} \left[g + (\rho+\theta g)(\sigma-1)(1-s_C) \right] \tag{21}$$

Equation (21) is a southern full employment condition that takes into account the implications of profit-maximizing R&D behavior by southern firms. Any increase in μ must be balanced by a corresponding decrease in δ_N to maintain equality in (21), given that g is pinned down by (15).[10] It follows that the southern steady-state condition is downward-sloping in (δ_N, μ) space: any decrease in the rate of imitation μ (which reduces both southern production and R&D employment) must be matched by an increase in relative R&D difficulty δ_N (which raises both southern production and R&D employment).

The northern and southern steady-state conditions are illustrated in Figure 1 and are labeled "North" and "South," respectively. Given that the northern steady-state condition is globally upward-sloping with a strictly positive δ_N intercept, the southern steady-state condition is globally downward-sloping with no intercepts, and the southern steady-state condition asymptotically approaches the δ_N axis, these two curves must have a unique intersection. Thus, the steady-state equilibrium values of δ_N and μ are uniquely determined.

Next we solve for asset holdings and consumer expenditures. Given that northern households own the firms located in the North and southern households own the firms located in the South, the aggregate assets in the North and the South are given by $\tilde{A}_{Nt} = V_{Nt} n_{Nt}$ and $\tilde{A}_{St} = V_{St} n_{St}$, respectively. Equations (9) and (17) imply that $\tilde{A}_{Nt} = w_N a_N (1-s_R) \delta_N L_{Nt} \gamma_N$, while (11) and (17) imply that $\tilde{A}_{St} = w_S a_S (1-s_C) \delta_N L_{Nt} \gamma_S (\gamma_S + k\gamma_N)^{-\theta}$. In steady-state, the individual budget constraints of consumers satisfy $\dot{\tilde{a}}_{it} = 0 = w_i + (\rho - g_L)\tilde{a}_{it} - c_i$ where $\tilde{a}_{it}$ represents the asset holding of the representative consumer in region i at time t. It follows that northern consumer expenditure is $c_N = w_N \left[1 + (\rho - g_L) a_N (1-s_R) \delta_N \gamma_N\right]$ and southern consumer expenditure is $c_S = w_S \left[1 + (\rho - g_L) a_S (1-s_C) \delta_N \gamma_S L_{N0} / [(\gamma_S + k\gamma_N)^{\theta} L_{S0}]\right]$.

To solve for the northern wage rate, we divide each side of (19) by the corresponding terms in (18) and use the definitions of $\bar{x}_{Nt}$ and $\bar{x}_{St}$ to obtain the *steady-state wage condition*

$$\frac{\left(\tau^{\sigma-1} + \eta\tau^{1-\sigma}\right)\frac{\gamma_N}{\gamma_S} + \left(\frac{w_N}{h}\right)^{\sigma-1}(1+\eta)}{\frac{(1+\eta)}{w_N}\left(\frac{w_N}{h}\right)^{1-\sigma}\frac{\gamma_N}{\gamma_S} + \frac{\eta\tau^{\sigma-1}+\tau^{1-\sigma}}{w_N}} = \frac{(g+\mu)^{\theta}(\rho+\theta g) a_S (1-s_C)}{(\rho+\mu+\theta g)(\mu+kg)^{\theta} a_N (1-s_R)} \tag{22}$$

[10] This can be seen by noting that $\frac{\partial}{\partial\mu}\{\mu(g+\mu)^{\theta-1}(\mu+kg)^{-\theta}\} = (g+\mu)^{\theta-1}(\mu+kg)^{-\theta}\{\theta\frac{kg}{\mu+kg} + (1-\theta)\frac{g}{g+\mu}\} > 0$.

where $\eta \equiv \frac{c_N L_{Nt}}{c_S L_{St}} = w_N \left(\frac{L_{N0}+(\rho-g_L)a_N(1-s_R)\delta_N\gamma_N L_{N0}}{L_{S0}+(\rho-g_L)a_S(1-s_C)\delta_N\gamma_S L_{N0}(\gamma_S+k\gamma_N)^{-\theta}} \right)$ is relative expenditure in the two regions. Since η/w_N does not depend on w_N and is completely pinned down by the previous steady-state equilibrium calculations, the denominator on the LHS of the wage equation is decreasing in w_N and the numerator is increasing in w_N. Hence the LHS of the wage equation is globally increasing in w_N. Furthermore, the LHS converges to zero as w_N converges to zero and the LHS converges to infinity as w_N converges to infinity. Thus, the wage equation (22) uniquely determines w_N.

Finally, we solve for consumer utility and the economic growth rate. Equation (2) implies that $u_{Nt} = [n_{Nt}(x_{Nt})^\alpha + n_{St}(x^*_{St})^\alpha]^{\frac{1}{\alpha}} = c_N/P_{Nt}$ and $u_{St} = [n_{Nt}(x^*_{Nt})^\alpha + n_{St}(x_{St})^\alpha]^{\frac{1}{\alpha}} = c_S/P_{St}$ where the price indexes for the two regions satisfy $(P_{Nt})^{1-\sigma} = n_t\left[\gamma_N(w_N/\alpha h)^{1-\sigma} + \gamma_S(\tau/\alpha)^{1-\sigma}\right]$ and $(P_{St})^{1-\sigma} = n_t\left[\gamma_N(\tau w_N/\alpha h)^{1-\sigma} + \gamma_S(1/\alpha)^{1-\sigma}\right]$. Taking logs and differentiating consumer utility yields $g_u \equiv \dot{u}_{Nt}/u_{Nt} = \dot{u}_{St}/u_{St} = g/(\sigma-1)$ Since utility is proportional in consumer expenditure holding prices fixed, utility growth equals real wage growth and we use it as our measure of economic growth. We conclude that the model has a unique steady-state equilibrium provided parameter values are such that the inequality $w_N > \tau h$ is satisfied.

5 Steady-State Properties of the Model

Under what conditions does the inequality $w_N > \tau h$ hold and a steady-state equilibrium exist? Unfortunately, we are not able to give a general answer to this question since the wage equation (22) is quite complicated. But there is a special case where the wage equation simplifies considerably, namely, when trade is costless ($\tau = 1$). In this section, we solve analytically for the model's steady-state properties assuming costless trade, since these calculations are particularly illuminating about how the model works. Then, in Section 6, we numerically solve the model for the main case of interest: when there are positive trade costs ($\tau > 1$).

In the special case of costless trade ($\tau = 1$), the wage equation simplifies considerably to

$$w_N = \left[\frac{(g+\mu)^\theta(\rho+\theta g)a_S(1-s_C)}{(\rho+\mu+\theta g)(\mu+kg)^\theta a_N(1-s_R)} \right]^{1/\sigma} h^{(\sigma-1)/\sigma}. \tag{23}$$

From solving the system of equations (20) and (21), we know that μ is a decreasing function of a_S. Since $\frac{\partial}{\partial\mu}\left[(\rho+\mu+\theta g)(\mu+kg)^\theta/(g+\mu)^\theta\right] > 0$, it follows from (23) that w_N is an increasing function of a_S. Also, as a_S converges to zero, w_N converges to zero and as a_S converges to infinity, w_N converges to infinity. Consequently, the equilibrium condition $w_N > h$ holds if and only if the parameter a_S exceeds a threshold level $\bar{a}_S$. We have established

Theorem 1 *When there is costless trade ($\tau = 1$), the model has a unique steady-state equilibrium if IPR protection in the South exceeds a threshold level ($a_S > \bar{a}_S$). Furthermore, the model can account for an arbitrarily large North-South wage ratio w_N/w_S if IPR protection is sufficiently strong (w_N/w_S converges to infinity as a_S converges to infinity).*

Theorem 1 is a key result and distinguishes this paper from the related literature. In earlier models with quality ladders and two-way product cycles, the North-South wage ratio must satisfy $\lambda > w_N/w_S > 1$ when there is costless trade, as we discussed in Section 2. In this paper by contrast, the model can generate an arbitrarily large North-South wage ratio w_N/w_S if IPR protection is sufficiently strong.[11]

To illustrate the model's implications, consider first the decision of a less-developed country to join the world trading system and thereby increase the size of the southern region ($L_{S0} \uparrow$). This change does not affect the northern steady-state condition (20) but implies that δ_N increases for given μ in (21), so the southern steady-state condition shifts to the right in (δ_N, μ) space as illustrated in Figure 1. Starting from the initial steady-state equilibrium, an increase in L_{S0} leads to an increase in both δ_N and μ. Now the measure of relative R&D difficulty $\delta_N \equiv n_t^{1-\theta}/L_{Nt}$ can only permanently increase if there is a temporary increase in the northern innovation rate $\dot{n}_t/n_t$. Also, it follows from (23) that w_N falls. We have established

Theorem 2 *When there is costless trade ($\tau = 1$), a increase in the initial size of the South ($L_{S0} \uparrow$) leads to (i) a permanent increase in the rate of imitation ($\mu \uparrow$), (ii) a temporary increase in the rate of innovation ($\delta_N \uparrow$), and (iii) a permanent decrease in the northern relative wage ($w_N/w_S \downarrow$).*

An increase in the initial size of the South naturally leads to more copying of northern products and this faster rate of technology transfer means that production (and jobs) move from the high-wage North to the low-wage South. With production jobs moving to the South, the northern relative wage must fall to make it attractive for northern firms to expand their R&D activities and to ensure that all the workers that lost their jobs in northern production are hired in northern R&D activities. In the short run, an increase in the initial size of the South causes the innovation rate

[11] Differences in R&D technologies is the key to why the model generates differences in wage rates across regions. We can see this most clearly by focusing on the special case where $h = 1$, $s_R = s_C = 0$ and $k = 1$. Then one unit of labor produces one unit of output in both regions and from (23), $w_N > w_S$ only holds when $a_S > a_N$. For northern workers to earn higher wages than southern workers, it must be easier for northern workers to innovate than it is for southern workers to imitate.

$\dot{n}_t/n_t$ to jump up and technological change to accelerate, but the innovation rate gradually falls back to the original steady-state level $g = g_L/(1-\theta)$ as R&D becomes relatively more difficult. In the long run, an increase in the initial size of the South does not change the innovation rate but increases relative R&D difficulty δ_N and the fraction of Northern labor employed in R&D activities [$a_N\delta_N g$ from (20)].

Second, consider the implications of an increase in the labor requirement associated with imitation ($a_S \uparrow$), which we interpret as stronger protection of intellectual property rights. This change has no effect on the northern steady-state equation (20) but implies that δ_N decreases for given μ in (21). Thus the southern steady-state condition shifts to the left in (δ_N, μ) space, decreasing both δ_N and μ. It follows from (23) that w_N rises. We have established

Theorem 3 *When there is costless trade ($\tau = 1$), an increase in intellectual property rights protection ($a_S \uparrow$) leads to (i) a permanent decrease in the rate of imitation ($\mu \downarrow$), (ii) a temporary decrease in the rate of innovation ($\delta_N \downarrow$), and (iii) a permanent increase the northern relative wage ($w_N/w_S \uparrow$).*

Stronger IPR protection makes it harder for southern firms to imitate northern products and the imitation rate naturally decreases. Varieties that previously would have been produced in the South are now produced in the North. This puts upward pressure on the northern wage and it must rise enough so that the increase in demand for northern production workers is completely offset by a decrease in demand for northern R&D workers, resulting in a lower northern innovation rate along the transition path to the new steady-state equilibrium. In negotiations about the protection of IPRs at the WTO, developing countries have been arguing that stronger IPR protection would simply generate substantial rents for northern innovators at the expense of southern consumers and would not stimulate faster technological change (see Maskus, 2000). Theorem 3 provides support for this position taken by developing countries.

Third, consider the implications of lower trade costs ($\tau \downarrow$). This change has no effect on the northern steady-state condition (20) and no effect on the southern steady-state condition (21). Thus, decreasing τ has no effect on either δ_N or μ. Totally differentiating the LHS of the wage equation (22) with respect to τ and w_N and then using the implicit function theorem, we obtain $dw_N/d\tau|_{\tau=1} = (\sigma-1)(\eta-1)w_N/\sigma(1+\eta)$. Increasing trade costs τ on the margin starting from costless trade increases the relative wage w_N if $\eta > 1$ and decreases the relative wage w_N if $\eta < 1$. We are mainly interested in the result going in the reverse direction

Theorem 4 *In the neighborhood of costless trade, a decrease in trade costs ($\tau \downarrow$) leads to (i) no change in the rate of imitation (μ constant), (ii) no change in the rate of innovation (δ_N constant), and (iii) an ambiguous impact on the northern relative wage $\frac{w_N}{w_S}$ [a permanent decrease if the North is larger than the South in terms of purchasing power ($c_N L_{Nt} > c_S L_{St}$) and a permanent increase if the North is smaller than the South in terms of purchasing power ($c_N L_{Nt} < c_S L_{St}$)].*

A key to understanding Theorem 4 is equation (5), which states that the profits of a northern firm only depend on its wage rate w_N and its level of production $\bar{x}_{Nt} L_t$. When the northern wage w_N increases, this increases both profits and R&D costs proportionately, and hence has no effect on the incentives of northern firms to innovate [w_N appears on both sides of (10) and hence cancels]. When the level of production $\bar{x}_{Nt} L_t$ increases, this raises firm profits and induces firms to employ more R&D workers [δ_N increases in (18)], but it is not possible for both northern production and R&D employment to increase, given that the same workers are used in both activities. Thus, a decrease in trade costs has no effect on northern production or R&D employment (and similar reasoning applied to the South yields that southern production and R&D employment are also unchanged).

A reduction in trade costs does, however, lead to a reallocation of resources in both the North and the South. Firms respond by exporting more, employing more workers to produce goods for the export market and employ fewer workers to produce goods for the domestic market. Lower trade costs mean that firms face stiffer competition in their domestic markets since the prices charged by other firms fall. For firms in the larger market, this stiffer domestic competition is more important in lowering labor demand than the increase in exporting is in raising labor demand, so lower trade costs tend to depress the relative wage of workers in the larger market.

Looking at the related literature, the main difference in results is Theorem 1. Dinopoulos and Segerstrom (2007), Parello (2007), and Sener (2006) present models of North-South trade with two-way product cycles and, as we discussed in section 2, these models have difficulty accounting for large North-South wage differences. When it comes to comparative steady-state properties, Dinopoulos and Segerstrom (2007) study the same three globalization-related events ($L_{S0} \uparrow$, $a_S \uparrow$, $\tau \downarrow$) and obtain the same results as in Theorems 2, 3 and 4. Parello (2007) analyzes a model with Cobb-Douglas consumer preferences and just studies the effects of stronger IPR protection but obtains the same results as in Theorem 3. Sener (2006) presents a model where economic growth is fully-endogenous and also just studies the effects of stronger IPR protection. He solves his model numerically and obtains results comparable to Theorem 3 except that the decrease in the rate of innovation is permanent.

6 Numerical Results

In this section, we report results obtained from solving the model numerically when there are positive trade costs ($\tau > 1$). In the computer simulations, we use the following benchmark parameter values: $\rho = 0.07$, $\alpha = 0.714$, $g_L = 0.014$, $L_{N0} = 1$, $L_{S0} = 2$, $\theta = 0.72$, $s_R = s_C = 0$, $a_N = 1$, $k = 0$, $\tau = 1.6$, $h = 1.25$, and $a_S = 2.94$. With these parameter choices, the steady-state economic growth rate is 2 percent, the market interest rate is 7 percent and North-South income differences are consistent with US-Mexico differences. The trade cost parameter $\tau = 1.6$ comes from Novy (2007), where the tariff equivalent of bilateral trade costs between the US and Mexico is estimated to be around 60 percent following the NAFTA free trade agreement in 1994.

The results from the computer simulations are reported in Table 1. The first column lists various endogenous variables that we solved for, the second column shows their steady-state equilibrium values given the benchmark parameters, the third column shows how the equilibrium values change when L_{S0} jumps up from 2 to 2.1, and the remaining columns show what happens when a_S is increased from 2.94 to 3.05, τ is decreased from 1.6 to 1.5, s_R is increased from 0 to 0.1 and s_C is increased from 0 to 0.1. There are two main conclusions that we draw from studying the results in Table 1.

First, for plausible parameter values, the model can account for large wage differences between the North and the South. In the benchmark parameter case (column 2), the northern wage rate is 121 percent higher than the southern wage rate ($w_N/w_S = 2.210$) even though we have assumed trade costs of 60 percent ($\tau = 1.6$). The only equilibrium condition for the existence of product cycles is easily satisfied ($w_N/w_S = 2.210 > \tau h = 2.000$). Even though we needed to assume sufficiently strong IPR protection ($a_S = 2.94$) to get a large North-South wage ratio w_N/w_S, a significant amount of production does move to the South. In the benchmark case, 19 percent of products are produced in the South ($\gamma_S = 1 - \gamma_N = 0.186$) and all of this production resulted from southern firms copying products developed in the North.

Second, the comparative steady-state properties that we derived analytically assuming costless trade continue to hold qualitatively when there are significant trade costs. All the insights that we derived in Section 5 assuming costless trade carry over to the more realistic case of significant trade costs. For example, as in Theorem 2, the increase in the size of the South ($L_{S0} \uparrow$) serves to decrease the northern relative wage (w_N/w_S falls from 2.210 to 2.122), and as in Theorem 3, stronger IPR protection ($a_S \uparrow$) serves to increase the northern relative wage (w_N/w_S rises from 2.210 to 2.296).

7 Concluding Comments

The model presented in this paper has several interesting comparative steady-state properties. When there is an initial jump in the size of the South (e.g., China joins the world trading system), we find that this stimulates both technology transfer to the South and innovation in the North, but also permanently reduces the relative wage of northern workers. When there is stronger IPR protection, making it harder for southern firms to copy northern technologies, we find that this retards technology transfer to the South and innovation in the North but permanently increases the relative wage of northern workers. We also study the effects of a permanent decrease in trade costs. Novy (2007) estimates that US trade costs with its major trading partners have declined on average from 83 percent in 1966 to 58 percent in 2002. We find that lower trade costs have no effect on the rates of technology transfer to the South and innovation in the North. But lower trade costs do lead to a permanent reduction in the relative wage of northern workers if the northern market is larger in terms of purchasing power.

To fully understand the basic properties of the model, we have restricted attention in this paper to the simple case where all technology transfer takes the form of southern firms copying northern products. In a companion paper, Gustafsson and Segerstrom (2008), we study how things change when technology transfer takes place within multinational firms. When foreign affiliates of northern-based multinational firms engage in adaptive R&D to learn how to produce their products in the low-wage South, then stronger IPR protection increases the rate of technology transfer to the South and increases the rate of innovation in the North. Furthermore, the relative wage of northern workers falls. Thus, the effects of stronger IPR protection are quite different when technology transfer takes place within multinational firms.

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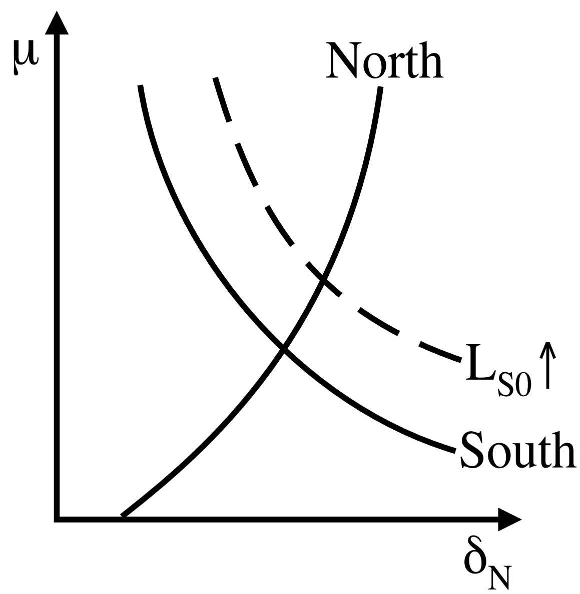

Figure 1: Steady-State Properties of the Model.

Table 1: Numerical results

	Benchmark	$L_{S0} \uparrow$	$a_S \uparrow$	$\tau \downarrow$	$s_R \uparrow$	$s_C \uparrow$
μ	0.011	0.013	0.010	0.011	0.008	0.015
δ_N	3.467	3.511	3.438	3.468	3.710	3.550
γ_N	0.814	0.788	0.831	0.814	0.854	0.766
w_N/w_S	2.210	2.122	2.296	2.208	2.438	2.003
τh	2.000	2.000	2.000	1.875	2.000	2.000
c_N	2.559	2.451	2.664	2.557	2.827	2.308
c_S	1.178	1.178	1.178	1.178	1.178	1.175
c_N/c_S	2.172	2.080	2.261	2.170	2.400	1.964