R&D Subsidies and Economic Growth

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Abstract

This paper presents an endogenous growth model in which some firms devote resources to developing higher quality products (innovative R&D) and other firms devote resources to copying these products (imitative R&D). Although consumers benefit from the knowledge created by both types of R&D activities, only innovative R&D subsidies lead to faster economic growth and imitative R&D subsidies actually lead to slower economic growth. A key assumption driving these conclusions is that R&D activities are subject to decreasing returns. When R&D activities are subject to constant returns, as is commonly assumed, the only equilibrium with both innovation and imitation is unstable.

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1 Introduction

It is now widely recognized that technological change is a major factor contributing to economic growth and that governments can influence the pace of technological change.[1] Many endogenous growth models have been developed in which the research and development (R&D) decisions of profit-maximizing firms determine the rate of technological change in the economy.[2] One of the main conclusions to emerge from this literature is that governments promote economic growth by subsidizing R&D expenditures.[3]

This paper examines the robustness of this conclusion. We develop a richer model of endogenous growth with two distinct types of R&D activities. Some firms engage in R&D to develop new, higher quality products (innovative R&D) whereas other firms engage in R&D to develop differentiated versions of other firms' products (imitative R&D).[4] Although both types of R&D activities create new knowledge which benefit consumers, we find that only innovative R&D subsidies lead to faster economic growth and imitative R&D subsidies actually lead to slower economic growth. Our analysis suggests that in countries where R&D is disproportionately imitative in nature, general R&D subsidies retard rather than enhance world economic growth.

Our modelling of economic growth is inspired by Schumpeter's (1942) description of "the process of creative destruction." In each industry, firms can freely enter into both innovative and imitative R&D races with other firms. Their behavior is determined by expected discounted profit maximizing considerations. The winner of each innovative R&D race learns how to produce a new superior quality product, and the winner of each imitative R&D race discovers how to produce a differentiated version of the state-of-the-art quality product (in its industry). Because successful innovators can price rival firms out of business, they earn monopoly profits. However, these monopoly profits are temporary, as a swarm of potential imitators strive to develop differentiated versions of each new product and successful imitation results in lower market prices. Thus, in a "gale of cre-

[1] See Grossman and Helpman (1994).

[2] Some early contributions to this literature include Romer (1990), Segerstrom, Anant and Dinopoulos (1990), Grossman and Helpman (1991a), and Aghion and Howitt (1992).

[3] See, for example, Romer (1990) and Grossman and Helpman (1991a).

[4] Given how quickly firms copy other firms' products, most of real world R&D may be imitative in nature. For empirical evidence on the rapid rate of imitation, see Mansfield, Schwartz and Wagner (1981), Mansfield (1985), and Caballero and Jaffe (1993). Mansfield, et. al. (1981) found that 60 percent of the patented innovations they studied were imitated by other firms within 4 years.

ative destruction," innovative firms drive established firms out of business, old products are replaced by new products and technical advance is diffused throughout the economy.

We show that if R&D activities are subject to sufficient decreasing returns, then the model has a unique steady state equilibrium, with positive rates of both innovation and imitation in each industry (Theorem 1). Focusing on this equilibrium, we find that innovative R&D subsidies increase innovation rates and imitative R&D subsidies increase imitation rates in each industry, exactly as one would expect. We are also able to pin down the cross effects. We find that imitative R&D subsidies decrease innovation rates, essentially because a faster rate of imitation makes the monopoly profits earned from successful innovation more short-lived. We also find that innovative R&D subsidies usually decrease imitation rates, mainly because faster technological change makes the profits from copying more short-lived. The only exception to this later conclusion occurs when the labor force is sufficiently small and copying is relatively inexpensive. In that case, the general equilibrium effects in the model are strong enough so that innovative R&D subsidies increase both innovation and imitation rates in each industry (Theorem 2).

To measure economic growth, we calculate the steady state utility growth rate for a representative consumer. We find that economic growth is positively related to the innovation rate and unaffected by the imitation rate in the economy. This property is somewhat surprising since imitation is not pure diffusion in our model but also involves knowledge creation. Consumers benefit from imitation because, due to product differentiation, they all prefer some "copied" products to the corresponding "original" products sold by innovative firms. They also benefit from imitation because increased product market competition results in lower market prices. Whereas innovation has growth effects, we find that imitation has only level effects on consumer utility. Thus, unlike in previous endogenous growth models, we find that not all forms of knowledge creation contribute to economic growth.

Given these properties of the model, the effects of R&D subsidies on economic growth follow immediately. Innovative R&D subsidies stimulate economic growth because they lead to a faster rate of innovation in each industry. More surprisingly, we find that imitative R&D subsidies retard economic growth. Even though they stimulate imitation, because this has only level effects on consumer utility, what really counts for growth is that imitative R&D subsidies lead to lower innovation rates in each industry. This cross effect drives our main result that imitative R&D subsidies retard economic growth (Theorem 3).

In the real world, R&D subsidies tend to be broad-based because it is not easy for governments to distinguish between innovative and imitative R&D. Thus, we also examine the effects of general R&D subsidies. Given the above mentioned results, a natural conjecture is that general R&D subsidies retard growth when equilibrium R&D effort is mainly imitative in nature and stimulate growth when equilibrium R&D effort is mainly innovative in nature. We find that this conjecture is false. Regardless of the equilibrium innovative/imitative R&D ratio, higher general R&D subsidies increase both innovation and imitation rates in each industry. Since general R&D subsidies stimulate innovative R&D effort, general R&D subsidies unambiguously promote economic growth (Theorem 4).

These theorems have interesting implications for the global economy. For the average country in the world, general R&D subsidies are likely to contribute to world growth. This holds even if R&D effort is mainly imitative in nature for the average country. For a developed country like the United States, where the innovative/imitative R&D ratio is presumably above average, general R&D subsidies are also likely to contribute to world growth. However, for developing countries like Korea which have specialized in imitative R&D, general R&D subsidies are, in effect, imitative R&D subsidies. Our analysis indicates that higher general R&D subsidies in such countries are likely to retard world economic growth. Higher imitation rates in these countries discourage innovative effort by firms in the rest of the world and it is the level of innovative R&D which determines the growth rate of the global economy.

There are two novel features of our model that help drive the above-mentioned results. First, we introduce a new way of modelling product differentiation to generate incentives for copying. The conventional method for modelling product differentiation is to assume that consumers have constant elasticity of subsitution (CES) preferences for variety.[5] Although many valuable insights have been generated using this approach, one drawback of assuming CES preferences is the implication that individual consumers buy every available product. This represents an extreme preference for variety in consumption. We adopt a fundamentally different (and simpler) approach by assuming preference diversity among consumers. Our assumptions about preferences imply that, in equilibrium, when there are two firms in an industry producing differentiated products, some consumers buy from the first firm, some consumers buy from the second firm, but no consumer buys from both. By producing differentiated products, each firms caters to a different clientele of consumers. Although highly sylized, our modelling of consumer preferences may capture better how product differentiation works in many industries. For the typical consumer, the set of products that are

[5]See, for example, Dixit and Stiglitz (1977), Helpman and Krugman (1985), and Romer (1990).

actually consumed is considerably smaller than the set of products that could be consumed.

The second novel feature of our model is the modelling of R&D activities. The conventional approach to modelling R&D in the Schumpeterian growth theory literature is to assume linear R&D technologies.[6] This "linear" assumption appears to have been made mainly for tractability reasons and has little to recommend itself on economic grounds. It implies that R&D behavior is infinitely sensitive to changes in the reward for R&D success: the slightest increase in the expected reward for winning a R&D race causes firms to choose infinitely high R&D intensities and the slightest decrease in the expected reward causes firms to stop doing R&D altogether. In this paper, we show that a tractable Schumpeterian growth model can be developed without assuming linear R&D technologies. Like most economic activities, we believe that R&D investment is subject to decreasing returns, and accordingly, we assume that there is a strictly concave (instead of linear) relationship between R&D employment and the instantanteous probability of R&D success.[7] With this assumption in place, we find that profit maximizing firms engage in more R&D when the expected reward is higher.

The question naturally arises, does changing this assumption make any difference (assuming concave instead of linear R&D technologies)? If the two assumptions generate qualitatively similar results, then a strong case can be made that the linear R&D assumption is more appropriate. After all, simplicity is a virtue in economic theory. To address this issue, we solve our model again in section 6 using linear R&D technologies and find that it makes a big difference.[8] The "linear R&D" model has a unique steady state equilibrium in which firms choose positive and finite levels of both innovative and imitative R&D, as was the case earlier. However, the comparative steady state properties of this equilibrium are perverse (innovative R&D subsidies lead to less innovative R&D and imitative R&D subsidies lead to less imitative R&D in each industry) and this equilibrium is unstable (Theorem 5). The "linear R&D" model has two other types of steady state equilibria which are stable but involve extreme R&D behavior (Theorem 6). In one of these equilibria, firms never do any imitative R&D and in the other equilibrium, firms always choose infinitely high imitative R&D

[6]See, for example, Barro and Sala-i-Martin (1995), Grossman and Helpman (1991a,b), Helpman (1992), Segerstrom (1991) and Taylor (1993). One exception is Aghion and Howitt (1992). They allow for non-linear R&D technologies but assume that all R&D is innovative in nature.

[7]For empirical evidence of decreasing returns, see Kortum (1993) and Thompson (1995).

[8]The same linear innovative and imitative R&D technologies were used in Grossman and Helpman (1991b) and Segerstrom (1991).

intensities. We conclude that the conventional "linear R&D" assumption has extreme implications and is worth avoiding in future research.

The remainder of the paper is divided into six sections. In section 2, we present our model of innovation and imitation, and in section 3, we show that it has a unique steady state equilibrium with appealing comparative steady-state properties. Section 4 explores the growth implications and section 5 explores the welfare (or public policy) implications of this model. In section 6, we demonstrate the importance of assuming significantly decreasing returns to R&D effort by solving the model again with linear R&D technologies. The model then has steady-state equilibria involving extreme R&D behavior. We also explore the implications of assuming slightly decreasing returns to R&D effort using computer simulations. Finally, in section 7 we discuss the related literature on innovation and imitation.

2 The Model

Consider an economy with a continuum of industries indexed by $\omega \in [0, 1]$. In each industry, firms are distinguished by the quality of the products they produce. We use the index j to measure quality. Higher values of j denote higher quality and j is restricted to taking on integer values. At time $t = 0$, the state-of-the-art quality product in each industry is $j = 0$, that is, some firm in each industry knows how to produce a $j = 0$ quality product and no firm knows how to produce any higher quality product. To learn how to produce higher quality products, firms in each industry engage in innovative R&D races. In general, when the state-of-the-art quality in an industry is j, the next winner of a innovative R&D race learns how to produce a $j + 1$ quality product. At the same time that some firms are trying to innovate, other firms try to copy state-of-the-art quality products. The winner of a imitative R&D race becomes a second "quality leader" in its industry.

2.1 Consumer Preferences

All consumers live forever and maximize discounted utility

$$U \equiv \int_0^\infty e^{-\rho t} u_i(t) dt \tag{1}$$

subject to the usual intertemporal budget constraint. In (1), ρ is the common subjective discount rate, and $u_i(t)$ is the consumer's static utility at time t. Consumers can be divided into two classes ($i = 1, 2$). For 50% of consumers (in terms of aggregate wealth), their static utility function takes the form

$$u_1(t) \equiv \int_0^{1/2} \log\left\{\sum_j \lambda^j \left[o(j,\omega,t) + \mu \cdot c(j,\omega,t)\right]\right\} d\omega$$
$$+ \int_{1/2}^{1} \log\left\{\sum_j \lambda^j \left[\mu \cdot o(j,\omega,t) + c(j,\omega,t)\right]\right\} d\omega \quad (2)$$

and for the other 50% of consumers, their static utility function takes the form

$$u_2(t) \equiv \int_{1/2}^{1} \log\left\{\sum_j \lambda^j \left[o(j,\omega,t) + \mu \cdot c(j,\omega,t)\right]\right\} d\omega$$
$$+ \int_0^{1/2} \log\left\{\sum_j \lambda^j \left[\mu \cdot o(j,\omega,t) + c(j,\omega,t)\right]\right\} d\omega, \quad (3)$$

where $o(j,\omega,t)$ denotes the quantity consumed of an original product of quality j produced in industry ω at time t, $c(j,\omega,t)$ denotes the quantity consumed of a copied product of quality j produced in industry ω at time t, $\lambda > 1$ is a measure of quality upgrading, and $\mu > 1$ is a measure of product differentiation.

Although these static utility functions seem rather complicated at first, they actually have simple economic interpretations. Suppose for the moment that $\mu = 1$. Then all consumers have identical preferences, exactly as in Grossman and Helpman (1991b) and Segerstrom (1991). The bracketed term $[o(j,\omega,t) + c(j,\omega,t)]$ in each consumer's utility function means that original and copied products of the same quality level j are perfect substitutes and generate identical utility per unit consumed. Since $\lambda > 1$ and λ^j increases in j, higher quality products generate more utility per unit consumed for each consumer.

Next consider how consumer preferences change when the preference diversity paremeter μ is increased. All the consumers continue to prefer products with higher j values (higher quality products), other things being equal. But now, original and copied products are not equivalent. Consider, for example, a consumer in the first class buying any state-of-the-art quality product in any industry $\omega \in [0, 1/2]$. The relevant bracketed term $[o(j,\omega,t) + \mu c(j,\omega,t)]$ indicates that price differences aside, this consumer prefers the copied to the original product. The same consumer in industries $\omega \in (1/2, 1]$ prefers original to copied products and preferences are reversed for consumers in the second class. Thus, with $\mu > 1$, 50% of consumers prefer original to copied products, the other 50% of consumers prefer copied to original products, and each consumer's preference depends on the industry ω under consideration. In the model, this preference diversity is needed to guarantee that copying firms earn positive economic profits and that copying occurs in equilibrium.

At each point in time t, each consumer allocates expenditure $E(t)$ to maximize $u_i(t)$ (i=1 or 2) given the prevailing market prices. Solving this optimal control problem yields a unit elastic demand function ($d = E(t)/p$ where d is quantity demanded and p is the relevant market price) for the product in each industry with the lowest quality adjusted price. The quantity demanded for all products with higher quality adjusted prices is zero.

Given this static demand behavior, each consumer chooses expenditures $E(t)$ over time to maximize U subject to an intertemporal budget constraint. Solving this optimal control problem yields the usual intertemporal optimization condition

$$\frac{\dot{E}(t)}{E(t)} = r(t) - \rho. \tag{4}$$

where $r(t)$ is the market interest rate at time t. In any steady state equilibrium, the left-hand side of (4) is zero, so the market rate of interest must equal the subjective discount rate at each moment in time. The actual level of expenditure E depends on the consumer's steady state assets.

2.2 Product Markets

Labor is the only input used in production and there are constant returns to scale. The labor market is perfectly competitive and for convenience, we normalize the wage of labor to equal one throughout time. Thus each firm has a constant marginal cost of production. For both quality leaders (firms that have won innovative or imitative R&D races and as a result, currently produce state-of-the-art quality products) and *quality* followers (firms that currently produce less than state-of-the-art quality products), one unit of labor is required to produce one unit of ouput. It follows that for both quality leader and quality follower firms, marginal cost equals one.

Every time a firm innovates and becomes a new quality leader, we assume that some technology diffusion occurs. Other firms learn how to produce the new product; however, not as cheaply as the current quality leader. These other firms (which we call *cost* followers) can produce the same product as the current quality leader but have a higher marginal cost of production $\sigma > 1$. Similarly, each time a firm imitates and becomes the second quality leader in an industry, we assume that further technology diffusion occurs. The cost followers learn how to produce both quality leaders' products more cheaply, that is, each cost follower's marginal cost of production drops from σ to $\phi < \sigma$. However, cost followers still cannot produce as cheaply as the quality leaders ($\phi > 1$).

In each industry, firms compete in prices and we solve for static Nash equilibrium behavior. We assume that $\lambda > \mu$ and $\lambda/\mu \geq \sigma$. These parameter conditions imply that when all firms practice marginal cost pricing, quality leaders are charging the lowest quality adjusted prices in each industry (all consumers buy from quality leaders) and their closest competitors are cost followers.

First, we solve for the Nash equilibrium in any industry where there is one quality leader. With all cost followers charging σ and all quality followers charging one, the lowest prices they can charge and not lose money, the quality leader earns the profit flow $\pi(p) = (p-1)E/p$ from charging the price p if $p \leq \sigma$, and zero profits otherwise (where E now represents aggregate consumer expenditure).[9] These profits are maximized by choosing the limit price $p = \sigma > 1$. Thus the quality leader earns the profit flow

$$\pi^L \equiv \left(\frac{\sigma - 1}{\sigma}\right) E \tag{5}$$

and none of the other firms in the industry can do better than break even (by selling nothing at all). This is the only Nash equilibrium outcome for industries where there is one quality leader.

Second, we solve for the Nash equilibrium in any industry where there are two quality leaders. Due to the assumption of preference diversity ($\mu > 1$), consumers are not in agreement about the relative quality of the two quality leaders' products. 50% of consumers prefer the copied product by the factor $\mu > 1$ and the other 50% of consumers prefer the original product by the same factor. Ignoring for the moment both cost and quality followers, if both quality leaders charge the same price p, then they both get 50% of consumers and each firm earns profits $\pi = (p-1)E/2p$. Since π is increasing in p and this holds even if the two leaders charge slightly different prices, clearly each quality leader wants to charge the highest price it can get away with, without losing its consumers to rival firms. The highest common price that the quality leaders can charge without losing all their customers to cost followers is the limit price ϕ. Then each quality leader earns the profit flow

$$\pi^C \equiv \left(\frac{\phi - 1}{2\phi}\right) E \tag{6}$$

(from 'copying' or being 'copied'). At the common price ϕ, the assumption $\lambda/\mu \geq \sigma$ implies that quality followers are priced out of business. To verify that this is a Nash equilibrium, it suffices to check that neither quality leader can gain by undercutting the other quality leader's price. With the other quality leader charging ϕ, a deviant quality leader can attract all 100% of consumers by charging the lower price ϕ/μ, and earn the deviant profit flow $(\frac{\phi}{\mu} - 1)\frac{E}{\phi/\mu}$. We assume that $2\mu - 1 \geq \phi$, to guarantee that neither quality leader gains by deviating.[10] Then both quality leaders

[9] As is the case in the rest of the quality ladders' literature, to break ties, we assume that when consumers are indifferent between buying from quality leaders and other firms, they choose to buy from the quality leaders.

[10] The parameter restrictions $\lambda/\mu \geq \sigma$ and $2\mu - 1 \geq \phi$ are both satisfied if σ and ϕ are sufficiently close to one, that

charging the limit price ϕ and earning the profit flow π^C represents the only Nash equilibrium outcome when there are two quality leaders.

Several properties of equilibrium behavior in product markets are worth noting. First, when copying occurs and an industry goes from having one to two quality leaders, the equilibrium price drops from σ to ϕ. Second, only quality leaders produce in equilibrium. Indeed, the only role that cost followers play in our analysis is to constrain the quality leaders from charging higher prices.[11] Given that consumer demand functions are unit elastic, quality leaders would like to charge infinitely high prices in the absence of competition from other firms. Third, firms earn positive economic profits from copying other firms' products and becoming quality leaders ($\phi > 1$ implies that $\pi^C > 0$). The key assumption driving this property is the assumption of preference diversity ($\mu > 1$). Without preference diversity ($\mu = 1$), copying a quality leader's product would result in a Bertrand equilibrium with zero profits for both quality leaders. No firm can justify doing costly imitative R&D with such a reward. Finally, in formulating consumer preferences, we have not allowed for a third differentiated product in each industry, so copying does not occur in industries that already have two quality leaders.

2.3 R&D Races

Two inputs are used to do innovative R&D in any industry, labor and a specialized input. Two inputs are also used to do imitative R&D in any industry, labor and a second specialized input. Whereas labor is perfectly mobile across sectors and industries, and can be used in either production, innovative R&D or imitative R&D, both specialized inputs are industry-specific and perfectly immobile.[12] We let L denote the endowment of labor in the economy and let F and G denote the endowments of the two specialized inputs in each industry. All input markets are perfectly competitive.

A firm i that hires ℓ_i units of (innovative R&D) labor and f_i units of the first specialized input in

is, there is enough technology diffusion. If these parameter restrictions are not satisfied, then existence of a pure strategy Nash equilibrium in prices is not guaranteed when there are two quality leaders. These parameter restrictions allow us to avoid analyzing complicated mixed strategy Nash equilibria.

[11] Since cost followers do not produce in equilibrium, in the rest of this paper, "copying" will always refer to firms becoming quality leaders by developing slightly differentiated versions of other quality leaders' products.

[12] We think of the two specialized factors as being workers that have special talents (or training) for doing innovative and imitative R&D, respectively. Since it takes a fundamentally higher skill level to innovate than to copy, we assume that workers with special talents (or training) for doing imitative R&D (the second specialized input) are not capable of doing innovative R&D.

industry ω at time t is successful in discovering the next higher quality product with instantaneous probability $A\ell_i^\alpha f_i^{1-\alpha}$. That is, $A\ell_i^\alpha f_i^{1-\alpha} \cdot dt$ is the probability that the firm will innovate by time $t + dt$ conditional on not having innovated by time t (dt is an infinitesimal increment of time). Likewise, a firm i that hires ℓ_i units of (imitative R&D) labor and g_i units of the second specialized input in industry ω at time t is successful in copying the quality leader's product in its industry with instantaneous probability $B\ell_i^\alpha g_i^{1-\alpha}$. The R&D parameters satisfy the conditions $A > 0$, $B > 0$ and $0 < \alpha < 1$. The returns to engaging in both innovative and imitative R&D races are independently distributed across firms, across industries, and over time.[13]

The savings of consumers are supplied to firms that engage in R&D through a capital market with the interest rate r adjusting to clear the market at each moment in time. Each firm issues a risky security that yields a positive return if it wins a R&D race and a negative return if it loses. Since there is a continuum of industries and the returns to engaging in R&D are independently distributed both across firms and industries, by holding a diversified portfolio of securities, investors are able to completely diversify away risk. Thus free entry into R&D implies that firms keep on entering each race until expected discounted profits are driven to zero.

Since all firms face the same factor prices and the innovative R&D "production function" is strictly quasi-concave and homogenous of degree one, in equilibrium, all innovative R&D firms must choose the same ℓ_i/f_i input ratio. This makes for convenient aggregation. The industry-wide instantaneous probability of innovative success becomes $I \equiv AL_I^\alpha F^{1-\alpha}$, where $\sum_i \ell_i = L_I$ is the industry-wide employment of innovative R&D labor and $\sum_i f_i = F$ is the fixed supply of the first specialized input. Using similar reasoning, the industry-wide instantaneous probability of imitative success becomes $C \equiv BL_C^\alpha G^{1-\alpha}$, where $\sum_i \ell_i = L_C$ is the industry-wide employment of imitative R&D labor and $\sum_i g_i = G$ is the fixed supply of the second specialized input. Since these are strictly concave functions of L_I and L_C respectively, all R&D is subject to decreasing returns at the industry level.

We want to solve the model for a steady state equilibrium where aggregate consumer expenditure E is constant over time, the industry R&D employment does not vary during any R&D race, the industry innovative R&D employment $L_I > 0$ is the same in all innovative R&D races, and the industry imitative R&D employment $L_C > 0$ is the same in all imitative R&D races. This is what

[13] Aghion and Howitt (1992) also assume that specialized factors are used in R&D but do not allow firms to copy other firms' products. They assume that there is only one specialized factor, instead of different specialized factors in different industries.

we mean by an *interior* steady state equilibrium.

Let v_I and v_C denote the expected discounted rewards for winning innovative and imitative R&D races respectively. By winning a imitative R&D race, a firm earns the copying profits π^C until it is driven out of business by further innovation. Thus, the reward for winning a imitative R&D race is

$$v_C \equiv \frac{\pi^C}{\rho + I}. \tag{7}$$

Copying profits are discounted using the equilibrium interest rate ρ and the instantaneous probability I that the firm will be driven out of business by further innovation. By winning an innovative R&D race, a firm earns the profits π^L until either (i) it is driven out of business by further innovation, or (ii) it is copied by another firm. Thus, the reward for winning a innovative R&D race is

$$v_I \equiv \frac{\pi^L + C\left[\frac{\pi^C}{\rho+I}\right]}{\rho + I + C}. \tag{8}$$

Leader firm profits are discounted using the equilibrium interest rate ρ, the instantaneous probability that the firm will be driven out of business by further innovation I, and the instantaneous probability that the firm will be copied C. If the firm is copied, then it earns profits at the lower rate π^C until it is driven out of business by further innovation.

At each moment in time t, a firm i that engages in a innovative R&D race chooses it's inputs ℓ_i and f_i to maximize its expected profits $v_I A \ell_i^\alpha f_i^{1-\alpha} - (1 - s_I)(\ell_i + w_F f_i)$, where w_F is the market wage for the first specialized input and the wage of labor has been normalized to equal one. We allow for the possibility that the government either subsidizes or taxes innovative R&D expenditures: $s_I \in (0, 1)$ if innovative R&D is being subsidized and $s_I \in (-\infty, 0)$ if R&D is being taxed. Partially differentiating expected profits with respect to ℓ_i, we obtain the first order condition $v_I \alpha A(f_i/\ell_i)^{1-\alpha} = 1 - s_I$. Since all firms face the same factor prices, they all chooses the same f_i/ℓ_i input ratio, and we can rewrite this first order condition as $v_I \alpha A(F/L_I)^{1-\alpha} = 1 - s_I$, or using $L_I = I^{1/\alpha} A^{-1/\alpha} F^{(\alpha-1)/\alpha}$ and (8), as

$$\frac{\pi^L + C\left[\frac{\pi^C}{\rho+I}\right]}{\rho + I + C} = \frac{(1 - s_I) I^{(1-\alpha)/\alpha}}{\alpha A^{1/\alpha} F^{(1-\alpha)/\alpha}}. \tag{9}$$

Using similar reasoning, the corresponding first order condition for maximizing expected profits in imitative R&D races is, using $L_C = C^{1/\alpha} B^{-1/\alpha} G^{(\alpha-1)/\alpha}$ and (7),

$$\frac{\pi^C}{\rho + I} = \frac{(1 - s_C) C^{(1-\alpha)/\alpha}}{\alpha B^{1/\alpha} G^{(1-\alpha)/\alpha}}, \tag{10}$$

where $s_C \in (-\infty, 1)$ is the rate at which the government subsidizes/taxes imitative R&D expenditures. Note that for both types of R&D races, the profit-maximizing intensity of R&D effort is an increasing function of the reward for winning, and just as long as the reward is positive, firms will undertake some R&D investment.

2.4 The Labor Market

At any point in time t, there is a proportion γ of industries which only have one quality leader and a proportion $1-\gamma$ of industries which have two quality leaders. We will refer to these simply as γ and $1-\gamma$ industries, respectively. When innovation occurs, the industry becomes a γ industry, and when imitation occurs, the industry becomes a $1-\gamma$ industry. During a time interval dt, an imitative R&D race in an γ industry ends and the industry becomes a $1-\gamma$ industry with probability Cdt. Likewise, during a time interval dt, an innovative R&D race in an $1-\gamma$ industry ends and the industry becomes an γ industry with probability Idt. This pattern is illustrated schematically in Figure 1. Thus in a steady state equilibrium where γ is constant over time, we must have $\gamma C = (1-\gamma)I$, or

$$\gamma = \frac{I}{I+C}. \tag{11}$$

In each γ industry, E/σ workers are employed in production and in each $1-\gamma$ industry, E/ϕ workers are employed in production. More workers are employed in production in $1-\gamma$ industries because the market price drops whenever copying occurs, and consumers respond by buying more in industries with two quality leaders. In the γ industries, L_C workers are employed in imitative R&D and in all the industries, L_I workers are employed in innovative R&D. Thus, full employment of labor implies that $\gamma\frac{E}{\sigma} + (1-\gamma)\frac{E}{\phi} + L_I + \gamma L_C = L$. Substituting for γ using (11), and for L_I and L_C, we obtain the economy's full employment of labor condition

$$\left[\frac{I}{I+C}\frac{1}{\sigma} + \frac{C}{I+C}\frac{1}{\phi}\right]E + \frac{I^{1/\alpha}}{A^{1/\alpha}F^{(1-\alpha)/\alpha}} + \frac{I}{I+C}\cdot\frac{C^{1/\alpha}}{B^{1/\alpha}G^{(1-\alpha)/\alpha}} = L \tag{12}$$

Given the R&D parameter restriction $0 < \alpha < 1$, the above described model can only be solved analytically if α takes on the middle-of-the-road value $\frac{1}{2}$, that is, R&D effort is subject to significant decreasing returns at the industry level. Thus, in exploring the steady state properties of the model, we will assume that $\alpha = \frac{1}{2}$ throughout the next two sections. Computer simulations of the more general model reveal that the properties derived assuming that $\alpha = \frac{1}{2}$ hold for a wide range of α values, indeed, they only break down when α is very close to one. To help shed light on why this is the case, in section 6, we present new steady state equilibrium calculations assuming that $\alpha = 1$ (the linear R&D model) and report some of the computer simulation results for other α values.

3 Steady State Equilibrium Properties

The model reduces to three steady state equations (9), (10), and (12) in three unknowns E, I, and C [using (5) and (6)]. We will now use these three equations to solve for three steady state equilibrium conditions in (C,I) space.

First, solving (10) for E [using (6)] and then substituting into (9) [using (5)] implicitly defines the *mutual R&D condition* in (C,I) space:

$$2aI(1-s_I)(\rho+I+C)=\left(\frac{\sigma-1}{\sigma}\frac{2\phi}{\phi-1}(\rho+I)+C\right)2bC(1-s_C), \tag{13}$$

where $a\equiv A^{-2}F^{-1}$ and $b\equiv B^{-2}G^{-1}$.

Second, solving (10) for E [using (6)] and then substituting into (12) yields the *imitative R&D condition* in (C,I) space:

$$\left(\frac{I}{I+C}\frac{1}{\sigma}+\frac{C}{I+C}\frac{1}{\phi}\right)(1-s_C)2bC(\rho+I)\frac{2\phi}{\phi-1}+aI^2+\frac{bIC^2}{I+C}=L \tag{14}$$

Third, solving (9) for E [using (6) and (5)] and then substituting into (12) yields the *innovative R&D condition* in (C,I) space:

$$\left(\frac{I}{I+C}\frac{1}{\sigma}+\frac{C}{I+C}\frac{1}{\phi}\right)\frac{(1-s_I)2aI(\rho+I+C)}{\frac{\sigma-1}{\sigma}+\frac{C}{\rho+I}\frac{\phi-1}{2\phi}}+aI^2+\frac{bIC^2}{I+C}=L \tag{15}$$

Note that, because of the way these three steady state conditions are constructed, at any intersection of two of the steady state conditions, the third condition must also be satisfied.

Equations (13), (14), and (15) have a variety of properties which are established in Appendix A. First, the mutual R&D condition is a globally upward sloping function which goes through the origin in (C,I) space. Second, the imitative R&D condition is a globally downward-sloping function when the labor force is sufficiently large, and is backward bending otherwise, with unique strictly positive C- and I-intercepts. Third, the innovative R&D condition is a globally downward-sloping function, with a unique strictly positive I-intercept and no positive C-intercept.

These three steady state conditions are illustrated in Figure 2 when the labor force is relatively large, and in Figure 3 when the labor force is relatively small. Since the mutual R&D condition is globally upward sloping and the innovative R&D condition is globally downward sloping, we are guaranteed a unique interior intersection of these steady state curves, regardless of the size of the labor force.[14] We have established

[14] In Figure 3, we have illustrated this intersection on the upward sloping part of the imitative R&D condition, but this is not necessarily the case. Whether the intersection is on the upward or downward sloping part of the imitative R&D condition depends on whether copying is relatively cheap or expensive.

Theorem 1 *The model has a unique interior steady state equilibrium.*

The upward sloping property of the mutual R&D condition has a simple economic interpretation. When consumer expenditure E is higher, firms find it more attractive to imitate because the profit flow π^C is higher. When consumer expenditure E is higher, firms also find it more attractive to innovate because the profit flow π^L is higher. Thus, with firms maximizing profits in both types of R&D races, more innovation is associated with more imitation.

The downward sloping property of the imitative R&D condition has the following economic interpretation: When firms innovate at a faster rate (I increases), firms that copy other firm's products find themselves driven out of business by further innovation more quickly. To break even in imitative R&D, these firms must earn higher copying profits (π^C) while they are in business. This only happens when consumer expenditure E is higher. But higher consumer expenditure means that more labor is devoted to production and higher I implies that more labor is devoted to innovative R&D. Given that there is a fixed endowment of labor L in the economy, more innovating (higher I) must come at the expense of less copying (lower C).

The only exception to the above reasoning occurs when the labor force is sufficiently small. Then, when the rate of innovation is low and firms innovate at a faster rate (I increases starting from a low value of I), the proportion of γ industries in the economy (industries with one quality leader) increases significantly. Since the market price drops as a result of copying, for each level of consumer expenditure E, less is produced in the higher priced γ industries, and less workers are engaged in production in the economy when there are more γ industries. Thus an increase in the rate of innovation can free up labor to do more copying even when consumer expenditure rises. Our calculations reveal that this occurs and the imitative R&D condition has an upward sloping part when the labor force is sufficiently small.

Finally, consider the intuition behind the negative slope of the innovative R&D condition: When firms imitate at a faster rate (C increases), firms that innovate find their profits being eroded more quickly by copying firms. To break even in innovative R&D, these firms must earn higher leader profits (π^L) before they are copied. This only happens when consumer expenditure E is higher. But higher consumer expenditure means that more labor is devoted to production and higher C implies that more labor is devoted to imitative R&D. Given that there is a fixed endowment of labor L in the economy, more imitating (higher C) must come at the expense of less innovating (lower I).

This model has appealing comparative steady state properties. When the government increases the innovative R&D subsidy s_I on the margin, the mutual R&D condition shifts up and the innovative R&D condition shifts to the right (there is no effect on the imitative R&D condition). As illustrated in Figure 4 by the movement from A to B, a higher innovative R&D subsidy results in more innovating and less copying if the labor force is large. The increase in innovation is unambiguous but the decrease in copying is not. If the labor force is relatively small and the interior steady state equilibrium lies on the upward sloping part of the imitative R&D condition, then a higher innovative R&D subsidy results in more innovating and more copying.

When the government increases the imitative R&D subsidy s_C on the margin, the mutual R&D condition shifts down and the imitative R&D condition shifts to the right (there is no effect on the innovative R&D condition). As illustrated in Figure 5 by the movement from A to B, a higher imitative R&D subsidy results in more copying and less innovating. Both conclusions are unambiguous and apply whether the labor force is large or small.

The government can also influence R&D activity by altering its level of patent enforcement. In our model, changes in patent enforcement can be modeled by assuming that the government has some control over B, one of the parameters in imitative R&D technology. Stronger patent enforcement should make it more difficult to imitate a quality leader's product. Thus, since a firm successfully imitates the quality leader's product with an instantaneous probability of $B\ell_i^{\alpha} g_i^{1-\alpha}$, we assume that stronger patent enforcement reduces B. It is straightforward to show that a decrease in B shifts the mutual R&D condition up to the right and shifts the innovative R&D condition down to the left. It follows that stronger patent enforcement reduces the rate of copying. The impact on innovative activity is ambiguous.

We have established the following theorem:

Theorem 2 *(i) A increase in the imitative R&D subsidy s_C increases the intensity of copying C and decreases the intensity of innovating I in each industry.*
(ii) A increase in the innovative R&D subsidy s_I increases the intensity of innovating I in each industry. It also decreases the intensity of copying C in each industry when the economy's labor force L is sufficiently large, but increases the intensity of copying C in each industry when the economy's labor force L is sufficiently small and copying is relatively inexpensive.
(iii) Stronger patent enforcement (a reduction in B) decreases the intensity of copying C and can either increase or decrease the intensity of innovating I in each industry.

4 Economic Growth

In this section, we explore how R&D subsidies influence the economy's growth rate. We measure the growth rate of the economy by calculating the growth rate of consumer utility for a consumer with static utility function $u_1(t)$, given by (2). All our conclusions also hold for consumers with the static utility function $u_2(t)$ given by (3).

Since along any steady state equilibrium path, consumers only buy state-of-the-art quality products, (2) can be rewritten as

$$\begin{aligned} u_1(t) &= \int_0^1 \log \lambda^j d\omega + \int_0^{1/2} \log\left[o(j,\omega,t) + \mu \cdot c(j,\omega,t)\right] d\omega \\ &\quad + \int_{1/2}^1 \log\left[\mu \cdot o(j,\omega,t) + c(j,\omega,t)\right] d\omega \end{aligned} \tag{16}$$

where $j = j(\omega, t)$ is the state-of-the-art quality level in industry ω at time t.

The index j increases when firms are successful in innovating, and firms engage in innovative R&D in all industries thoughout time in any steady state equilibrium. For any industry ω, the probability of exactly m improvements in a time interval of length τ is $f(m,\tau) = [I\tau]^m e^{-I\tau}/m!$ Thus $f(m,\tau)$ represents the measure of products that are improved exactly m times in an interval of length τ. Using the properties of the Poisson distribution (see Hoel, Port and Stone (1971), page 84), it follows that the first integral in (16) equal

$$\sum_{m=0}^{+\infty} f(m,\tau)[\log \lambda^m] = tI \log \lambda \tag{17}$$

In any steady state equilibrium consumer expenditure E is constant over time. When there is one quality leader in an industry, $o(j,w,t) = E/\sigma$. When there are two quality leaders, $o(j,w,t) = 0$ and $c(j,w,t) = E/\phi$ for $\omega \in [0,1/2]$, whereas $o(j,w,t) = E/\phi$ and $c(j,w,t) = 0$ for $\omega \in (1/2,1]$. That is, the consumer prefers the copied product in industries $\omega \in [0,1/2]$ and prefers the original product in the remaining industries. Since the proportion of industries $\omega \in [0,1/2]$ with one quality leader and the proportion of industries $\omega \in (1/2,1]$ with one quality leader are both given by (11) in any steady state equilibrium, the second and third integrals in (16) are both constants over time. It follows that the growth rate of consumer utility g is obtained by differentiating (17) with respect to t:

$$g \equiv \frac{du_1(t)}{dt} = I \log \lambda. \tag{18}$$

What is surprising about (18) is that growth is not a function of the intensity of copying C in the economy. Growth increases when firms devote more resources to innovative R&D but copying has only level effects. For any given steady state innovation rate I, an increase in the steady state imitation rate C increases consumer utility immediately (because more industries with two quality leaders means lower prices for consumers and more differentiated products to choose from) but does not change the rate at which consumer utility grows over time (the proportion of industries with two quality leaders does not change in a steady state equilibrium and movement up the quality ladder is completely determined by the innovation rate in each industry). Indeed, in this model, copying by itself cannot sustain economic growth in the long run. With no innovating and only copying, consumer utility rises over time but eventually plateaus as firms run out of new products to copy.

Since innovative R&D subsidies increase I whereas imitative R&D subsidies decrease I, it immediately follows that

Theorem 3 *A higher innovative R&D subsidy s_I leads to faster economic growth, whereas a higher imitative R&D subsidy s_C leads to slower economic growth.*

It is worth noting that allowing for cross-R&D spillovers (success in imitative R&D increasing a firm's effectiveness in doing innovative R&D) changes our result that imitation has only level effects. However, such cross-R&D spillovers would have to be quite strong to reverse our main conclusion that imitative R&D subsidies retard economic growth. To obtain simpler, cleaner results and to facilitate comparison with the previous literature, we have left cross-R&D spillovers out of the model.

In the real world, R&D subsidies tend to be broad-based because it is not easy for governments to distinguish between innovative and imitative R&D. Thus, we also want to examine the effects of general R&D subsidies. Given the above mentioned results, a natural conjecture is that general R&D subsidies retard growth when equilibrium R&D effort is mainly imitative in nature and stimulate growth when equilibrium R&D effort is mainly innovative in nature. Surprisingly, this conjecture is false. Since an increase in $s = s_I = s_C$ shifts both the innovative and imitative R&D conditions to the right but has no effect on the upward sloping mutual R&D condition, both I and C increase. Thus regardless of the equilibrium innovative/imitative R&D ratio,

Theorem 4 *A higher general R&D subsidy $s = s_I = s_C$ increases both innovation and imitation rates in each industry and leads to faster economic growth.*

If R&D is mostly imitative (the slope of the mutual R&D condition is close to zero), then the dominant effect of an increase the general R&D subsidy rate is to increase the imitation rate in each industry. But because the mutual R&D condition is upward sloping, the innovation rate must also increase to some extent.

5 Policy Analysis

Even though growth is not a function of the intensity of copying (C), imitation does raise welfare by creating the knowledge required to produce a differentiated version of the state-of-the-art product. The introduction of this new product leads to lower prices and higher consumer welfare as 50% of the consumers prefer this new product to the initial one. It follows that policies that enhance growth may not be socially optimal if they significantly reduce the rate of of copying.

To get some handle on the tradeoffs between level effects and growth effects of R&D policies we calculate the optimal subsidy rates in two carefully chosen examples. Carrying out the integration in (16) yields

$$u_1(t) = tg + \gamma \log(E/\sigma) + (\gamma/2)\log(\mu) + (1-\gamma)\log(\mu E/\phi)$$

where g denotes the growth rate as given in (18). Substituting $u_1(t)$ into (1) and solving we obtain our measure of welfare:

$$U = g/\rho^2 + \{\log(E/\phi) - \gamma[\log(E/\sigma) - \log(E/\sigma)] + [1-(\gamma/2)]\log(\mu)\}/\rho$$

The positive influence of growth on welfare is captured by the first term while the influence of copying is more subtle, working through changes in γ. Increases in the rate of copying C reduce γ. This results in a steady-state with more two-leader industries, where the relatively low price of ϕ is charged, and fewer single-leader industries, where the higher price of σ is charged. The increase in welfare generated by this change in industry composition is captured by the third term. The fact that copying also results in a new product that some consumers prefer is captured by the last term. It follows that subsidies that enhance growth raise welfare by increasing the first term, but may lower welfare by reducing the last two terms if they result in less imitative activity.

To calculate the optimal subsidy rates we choose values for the parameters of our model and solve for the subsidy rates that maximize U. Assuming that our parameters are chosen wisely, this should give us some idea of the range in which the optimal subsidy rates fall. It also allows us to compare different types of R&D policies.[15] We begin by normalizing the total amount of the

[15]The results we derive below are based on comparing steady states. We return to this issue and discuss it in greater detail at the end of the section.

three factors available in the economy (L, F and G) to unity, with the vast majority of the factor endowment consisting of labor, the mobile factor. Thus, we set $L = .95$ and $F = G = .025$. The subjective discount rate ρ is set equal to .03, implying a 3% steady-state interest rate.

The remaining parameter values are set so that the model yields predictions consistent with the stylized facts in the industrial organization and growth literatures. In deriving our equilibrium we assumed that $\lambda/\mu \geq \sigma > \phi > 1$ and that $2\mu - 1 \geq \phi$. Values of the parameters that are consistent with these inequalities are $\lambda = 1.5$, $\mu = 1.1$, $\sigma = \lambda/\mu = 1.36$, and $\phi = 2\mu - 1 = 1.2$. These values imply that a single industry leader charges a mark-up of 36% and that two leaders charge a mark-up of 20%. Both values are consistent with results reported in the empirical literature on mark-up rates in concentrated industries (see, for example, Carleton and Perloff (1994)), where most studies find mark-up rates in the range of 10-50%.

The two remaining parameters are A and B. In our first example, these parameters are set so that new products are copied, on average, with a three year time lag ($C = .333$) and so that the growth rate is .5% per year ($g = .005$). This growth rate was chosen to be roughly consistent with Denison's (1985) finding that in the U.S. over the period 1929-1982 advances in knowledge contributed about .55% to the growth rate each year. The values consistent with $C = .333$ and $g = .005$ are $A = .602$ and $B = 3.49$. In our second example, we alter the values of A and B so that growth occurs much more rapidly. In particular, we set $A = 1.57$ and $B = 4.85$. These values lead to equilibrium growth and copying rates of 2% and .333, respectively. We consider this second example in order to see if the optimal policies for low-growth and high-growth economies differ in any significant respect.

We consider four different policies. First, we assume that the government can directly target subsidies toward innovative activity. Thus, we calculate the optimal value of s_I holding s_C fixed at zero. Next, we assume that the government can control both s_I and s_C and solve for the optimal subsidy for innovation and the optimal tax on copying. As we show below, this leads to the first-best outcome. We then turn to the case in which the government cannot distinguish between the two types of R&D and can therefore only set broad-based subsidy rates (so that $s_I = s_C = s$). We solve for the optimal broad-based subsidy when the government chooses the level of patent protection and also when the level of patent protection is held fixed.

Our results are reported in Table 1. For each example, the laissez-faire equilibrium is reported in the first row. For the cases in which the government can distinguish between types of R&D the optimal subsidies are listed in the second and third rows. The second row gives the optimal innovation subsidy when $s_C = 0$ and the third row gives the optimal subsidies and taxes when the government can control both s_I and s_C. The last two rows report the optimal subsidy rates when the government uses general R&D subsidies ($s_I = s_C$). In the fourth row the government does not alter its level of patent enfocement while in the last row patent enforcement is set at its optimal level.

Several conclusions can be drawn from these two examples. First, regardless of the type of policy, innovation should be subsidized at a fairly high rate. When subsidies can be targeted, the optimal innovation subsidy falls in the range of 75-90% depending on the equilibrium growth rate and whether or not the government taxes copying. Second, even though copying is beneficial in that it leads to lower prices and more knowledge, its negative impact on the rate of innovation is far more important. This results in fairly high taxes on copying when the government can distinguish between types of R&D and it results in extremely rigorous patent protection when only general subsidies are possible. In our first example, the optimal policy involves taxing copying at a rate of 150% which drives the rate of copying down to .064 (so that products are copied, on average, with a 15.6 year time lag). In the high-growth example, it is optimal to drive the rate of copying to zero.

The third conclusion that we can draw is that a general R&D subsidy combined with strong patent protection may be a second-best policy. The first-best outcome is always achieved by heavily subsidizing innovation and heavily taxing imitation. In the high-growth example, this same outcome can be achieved with a large general subsidy for R&D and extremely strong patent protection. However, in our main example, a general subsidy and strong patent protection is only the third-best policy choice leading to lower welfare than what could be achieved by only subsidizing innovation.

One problem with the results reported in Table 1 is that they are derived by comparing steady-states. We have not solved for the complicated transition paths between steady-states due to policy changes. In our steady-state welfare calculations, we assume that the state-of-the-art quality level is $j = 0$ in both $\gamma = \frac{I}{I+C}$ and $1 - \gamma = \frac{C}{I+C}$ industries at time $t = 0$. When C increases (holding I fixed), more industries start out with two quality leaders. Since consumers benefit from copying, a higher steady-state value of C means that the economy is technologically more advanced at time $t = 0$. Thus, our steady-state welfare calculations are biased and over-value copying. However, since we find that it is optimal to heavily subsidize innovative R&D and heavily tax imitative R&D, removing the bias would make our policy conclusions even stronger.[16]

[16]In order to get some handle on how biased our results are, we recalculated the optimal R&D subsidies using an alternative measure of welfare that is biased in the opposite direction. We instead assumed that the state-of-the-art quality level is $j = 1$ in $\gamma = \frac{I}{I+C}$ industries and is $j = 0$ in the remaining $1 - \gamma = \frac{C}{I+C}$ industries at time $t = 0$. Then a higher

6 The Linear R&D Model

Throughout the previous two sections, we have assumed significant decreasing returns to R&D at the industry level ($\alpha = \frac{1}{2}$). In this section, we explore the steady state implications of instead assuming constant returns to R&D ($\alpha = 1$). In the new "linear R&D model," both innovative and imitative R&D technologies are the same as in Grossman-Helpman (1991b) and Segerstrom (1991). Other than changing α, we make no other changes to the "square root R&D model" presented in section 2.

First, we substitute $\alpha = 1$ into (9), (10) and (12). Eliminating the variable E from these equations in the same manner as was described earlier yields a new *mutual R&D condition* in (C,I) space:

$$(\rho + I) \left\{ \frac{\sigma - 1}{\sigma} \frac{2\phi}{\phi - 1} (1 - s_C) b - a(1 - s_I) \right\} = C[a(1 - s_I) - b(1 - s_C)], \tag{19}$$

a new *imitative R&D condition* in (C,I) space:

$$\left\{ \frac{I}{I + C} \frac{1}{\sigma} + \frac{C}{I + C} \frac{1}{\phi} \right\} (1 - s_C) b(\rho + I) \frac{2\phi}{\phi - 1} + aI + \frac{ICb}{I + C} = L, \tag{20}$$

and a new *innovative R&D condition* in (C,I) space:

$$\left\{ \frac{I}{I + C} \frac{1}{\sigma} + \frac{C}{I + C} \frac{1}{\phi} \right\} \frac{(1 - s_I) a(\rho + I + C)}{\frac{\sigma - 1}{\sigma} + \frac{C}{\rho + I} \frac{\phi - 1}{2\phi}} + aI + \frac{ICb}{I + C} = L, \tag{21}$$

where now $a \equiv A^{-1}$ and $b \equiv B^{-1}$.

In order to guarantee that the linear R&D model has an interior steady state equilibrium, we need to introduce some new assumptions about parameter values. First, we will assume that firms can imitate more easily that innovate, taking into account R&D subsidies:

Assumption A1. $a(1 - s_I) > b(1 - s_C) > 0$.

The empirical work of Mansfield, et. al. (1981) supports such a ranking. They found a .65 average ratio of imitation to innovation costs. A1 implies that the right hand side of the mutual R&D

steady-state value of C (holding I fixed) means that the economy is technologically less advanced at time $t = 0$. The results derived using this alternative measure of steady-state welfare are remarkably similar to those shown in Table 1. With general subsidies (the last two rows), the optimal subsidies are identical to the fourth decimal place. With specific subsidies, the optimal innovation subsidy is slightly higher using the alternative measure of welfare [in the main example, $s_I = .865$ in the second row (as opposed to .821) and in the high growth example, $s_I = .911$ in the second row (as opposed to .911)]. Thus, ignoring the transition paths between steady-states does not appear to bias our results in a significant manner.

condition is non-negative for $C \geq 0$. Then the mutual R&D condition only intersects the positive orthant of (C,I) space if the relative profit (π^I/π^C) from innovating (versus imitating) exceeds the relative cost ($a(1-s_I)/b(1-s_C)$), that is,

Assumption A2. $\frac{\sigma-1}{\sigma}\frac{2\phi}{\phi-1} > \frac{a(1-s_I)}{b(1-s_C)}$.

Given A1 and A2, the mutual R&D condition is unambiguously upward sloping and linear in (C,I) space, with a strictly positive C-intercept.

Next consider the imitative R&D condition. It has no positive C-intercept provided we assume that the labor force is sufficently large, that is,

Assumption A3. $\frac{(1-s_C)2b\rho}{\phi-1} < L$.

This assumption is needed since otherwise, it is not profitable to do imitative R&D even when copying profits π^C are earned forever ($I = 0$). Given A1-A3, the imitative R&D condition has a unique strictly positive I-intercept. It is also a globally downward sloping function in (C,I) space (see the appendix).

Finally, consider the innovative R&D condition (21). It has a unique, strictly positive I-intercept only if the labor force is sufficently large,

Assumption A4. $\frac{(1-s_I)a\rho}{\sigma-1} < L$.

We will make this assumption since otherwise, it is not profitable to do innovative R&D even when there is no copying ($C = 0$). Given assumptions A2 and A3, the innovative R&D condition has a strictly positive C-intercept if and only if $2a\rho(1-s_I)/(\phi-1) > L$. We will consider both possibilities (a unique strictly positive C-intercept when L is relatively small and no positive C-intercept when L is relatively large). In either case, the innovative R&D condition is a globally downward sloping function in (C,I) space (see the appendix).

All three of the steady state equations are illustrated in Figure 6. As illustrated, the innovative R&D condition's I-intercept exceeds the imitative R&D condition's I-intercept. This is an unambiguous implication of assumption A2. The upward sloping, linear property of the mutual R&D condition together with the downward sloping property of the imitative R&D condition imply that these curves must have a unique intersection. Thus, the linear R&D model has a unique interior steady state equilibrium.

The slopes of the R&D conditions have the same economic interpretations as was given earlier for the square root R&D model. However, the linear R&D model has different comparative steady state properties. When the government increases the innovative R&D subsidy s_I on the margin, the mutual R&D condition shifts down and the innovative R&D condition shifts to the right (there is no effect on the imitative R&D condition). As illustrated in Figure 6 by the movement from A to B, a higher innovative R&D subsidy results in more copying and less innovating. Furthermore, when the government increases the imitative R&D subsidy s_C on the margin, the mutual R&D condition shifts up and the imitative R&D condition shifts to the left (there is no effect on the innovative R&D condition). As illustrated in Figure 7 by the movement from A to B, a higher imitative R&D subsidy results in more innovating and less copying. Normally, one would expect that a government policy of subsidizing an activity like innovative R&D would encourage firms to do more of that activity. But Figures 6 and 7 indicate that the opposite is the case. Innovative R&D subsidies lead to less innovating and imitative R&D subsidies lead to less imitating!

Given these perverse comparative steady state results, the question naturally arises: is this steady state equilibrium stable? To assess the stability properties of this steady state there are two paths that we can take. First, we could analyze the model's "equations of motion" to determine whether or not there exists a rational expectations saddlepath that would lead the economy back to this steady state if the economy deviated from it at some point in time. This approach assumes that agents are able to perfectly forecast the equilibrium dynamic adjustments that are required along the transition path to the steady state. Second, we could ask how the economy might evolve over time if it began in disequilibrium and attempt to determine if there is a sensible learning process that would lead the economy toward this equilibrium. For example, it seems natural to assume that if the reward to an activity increases that agents would respond by increasing their intensity of that activity. If so, would such a response lead the economy toward the equilibrium derived above? This second approach requires us to graft an ad hoc adjustment process onto the model in order to analyze its non-steady state behavior. It assumes that agents lack the information needed for perfect foresight and asks what might happen if they followed a simple but intuitively appealing adaptive rule of raising (lowering) the intensity of an activity whenever the cost was less (more) than the projected current benefit under static expectations.

In our opinion, for an equilibrium to be attractive, it should satisfy both types of stability. That is, not only should it be saddlepath stable in the formal sense, but there should also be a reasonable learning process that would lead the economy back toward that equilibrium if it ever deviated from the rational expectations saddlepath. Otherwise, we are left with questions such as "what would induce agents to raise their intensity of imitative activity when it is penalized more heavily, other than the foreknowledge that this is an equilibrium response?"

In Appendix B, we demonstrate that the steady state equilibrium derived above is indeed saddlepath stable. However, it fail the second test of stability. A phase diagram for the linear R&D model is illustrated in Figure 8, where we have assumed a standard adjustment process: when firms earn positive (negative) expected discounted profits from R&D investment, they respond by gradually increasing (decreasing) their R&D expenditures. Each point in this phase diagram can be interpreted as a possible state of the economy at a point in time. The coordinates of a point describe the currently prevailing innovation (I) and imitation (C) rates in each industry, with the level of consumer expenditure E being implicitly determined by (12). The $\dot{I} = 0$ curve coincides with the previously derived innovative R&D condition (21). At any point on this curve, innovative R&D firms are earning zero expected discounted profits and have no incentive to change their behavior. Since an increase in C lowers the expected discounted profits of innovative R&D firms, above the $\dot{I} = 0$ curve, innovative R&D firms are losing money, implying that $\dot{I} < 0$. Likewise, the $\dot{C} = 0$ curve coincides with the previously derived imitative R&D condition (20). Since an increase in I lowers the expected discounted profits of imitative R&D firms, above the $\dot{C} = 0$ curve, imitative R&D firms are losing money, implying that $\dot{C} < 0$. Using similar reasoning, we derive all the arrows of motion in Figure 8. It follows immediately that the interior steady state equilbrium given by point E_2 is unstable (assuming the standard adjustment process).[17] The above-mentioned arguments have established

Theorem 5 *Given A1-A4, the linear R&D model has a unique interior steady state equilibrium and it is unstable (assuming a standard adjustment process).*

Although the only steady state equilibrium with both innovation and imitation in the linear R&D model is unstable, Figure 8 suggests that the model has two other steady state equilibria that are stable. Two possible transition paths for the economy are illustrated, one converging over time to point E_1 and the other to point E_3. At point E_1, the imitation rate is zero and at point E_3, the imitation rate is positive infinity! We will now verify that the linear R&D model always has both types of steady state equilibria.

At each moment in time, a firm i that engages in innovative R&D chooses its innovative R&D labor input ℓ_i to maximize its expected profits $v_I \ell_i / a - \ell_i(1 - s_I)$ and a firm i that engages in

[17] Applying the same reasoning to the square-root R&D model analyzed in sections 3 and 4, we find that the interior steady state equilibrium is definitely stable in the "large world" case. However, determining stability of equilibrium in the "small world" case is more problematic and is beyond the scope of this paper. This equilibrium could be an unstable focus if the innovative R&D condition is sufficiently steep and the imitative R&D condition is sufficiently flat (in Figure 3).

imitative R&D chooses its imitative R&D labor input ℓ_i to maximize its expected profits $v_C\ell_i/b - \ell_i(1-s_C)$. If $v_I \geq a(1-s_I)$, then $\ell_i = +\infty$ is profit maximizing and if $v_I \leq a(1-s_I)$, then $\ell_i = 0$ is profit maximizing. We will use these insights below.

First, consider whether there exists a steady state equilibrium with $C = 0$ and $I > 0$. If so, then substituting $C = 0$, $I > 0$ and $\alpha = 1$ into (9), (10), and (12) [using (5) and (6)], we obtain three new steady state conditions:

$$\frac{(\phi-1)E}{2\phi(\rho+I)} \leq b(1-s_C). \tag{22}$$

$$\frac{(\sigma-1)E}{\sigma(\rho+I)} = a(1-s_I). \tag{23}$$

$$\frac{E}{\sigma} + aI = L \tag{24}$$

The first order condition (22) must hold with an inequality for the corner solution $C = 0$ to be profit maximizing. Assumption A4 guarantees that (23) and (24) have a unique $I > 0$, $E > 0$ solution and assumption A2 guarantees that this solution satisfies (22) with a strict inequality. Thus, an equilibrium with no copying exists, given A2 and A4. This equilibrium with no copying is the steady state equilibrium Grossman and Helpman (1991a) solved for and exclusively analyzed. As we have shown, it exists under precisely the assumptions that were used to guarantee the existence of a interior steady state equilibrium with both copying and innovating.

Second, consider whether there exists a steady state equilibrium with $C = +\infty$ and $I > 0$. If so, then substituting $C = +\infty$ and $I > 0$ into (9), (10), and (12) [using (5) and (6)], we obtain the three steady state conditions:

$$\frac{(\phi-1)E}{2\phi(\rho+I)} \geq b(1-s_C). \tag{25}$$

$$\frac{(\phi-1)E}{2\phi(\rho+I)} = a(1-s_I). \tag{26}$$

$$\frac{E}{\phi} + (a+b)I = L \tag{27}$$

The first order condition (25) must hold with an inequality for the corner solution $C = +\infty$ to be profit maximizing. Assumption A1 guarantees that any solution to (26) and (27) satisfies (25) with a strict inequality. We obtain a $I > 0$, $E > 0$ solution to (26) and (27) if and only if

$$L > \frac{2\rho a(1-s_I)}{\phi-1}. \tag{28}$$

Any parameter values that satisfy (28) also satisfy assumption A4 since $\sigma > \phi$. Thus the model has a $C = +\infty$, $I > 0$ steady state equilibrium if the labor force L is sufficiently large.

Finally, consider whether there exists a steady state equilibrium with $C = +\infty$ and $I = 0$. If so, then substituting $C = +\infty$ and $I = 0$ into (9), (10), and (12) [using (5) and (6)], we obtain the three steady state conditions:

$$\frac{(\phi-1)E}{2\phi\rho} \geq b(1-s_C). \tag{29}$$

$$\frac{(\phi-1)E}{2\phi\rho} \leq a(1-s_I). \tag{30}$$

$$\frac{E}{\phi} = L \tag{31}$$

Substituting (31) for E in (29) and(30), we obtain the double inequality

$$\frac{2\rho b(1-s_C)}{\phi-1} \leq L \leq \frac{2\rho a(1-s_I)}{\phi-1} \tag{32}$$

Whenever assumption A3 is satisfied, either (32) or (28) holds. Thus, a steady state equilibrium with $C = +\infty$ always exists. Whether $I = 0$ or $I > 0$ depends on the size of the labor force, and our assumptions A1-A4 allow for both possibilities. We have established

Theorem 6 *Given A1-A4, in addition to the interior steady state equilibrium, the linear R&D model always has a $I > 0, C = 0$ steady state equilibrium and a $I \geq 0, C = +\infty$ steady state equilibrium.*

The linear R&D model has surprising and disturbing steady state properties. The only equilibrium with both innovation and imitation has perverse comparative steady state properties and is unstable (in at least one important sense). If the economy is initially on this steady state equilibrium path, the slightest increase in the innovative R&D subsidy stimulates innovative R&D and makes any effort devoted to imitating unprofitable. On the other hand, the slightest decrease in the innovative R&D subsidy makes any effort devoted to innovating unprofitable given the prevailing imitation rate in the economy. Theorem 6 indicates that we should observe either one of two possibilities: the quality of products being improved so rapidly that firms find it unprofitable to imitate other firms' products, or the copying of other firms' products occurring so rapidly that firms find it unprofitable to innovate. Since we *do* observe innovation and rapid imitation of new products by other firms, this is indirect evidence that the key assumption driving Theorems 5 and 6, constant returns to R&D, is too strong. Our preferred model of innovation and imitation is the model studied in sections 3-5 with significant decreasing returns to R&D at the industry level.

Given that the general model introduced in section 2 has fundamentally different steady state properties when $\alpha = \frac{1}{2}$ and $\alpha = 1$, the question naturally arises: what happens for intermediate values of α? We have investigated this issue using computer simulations of the general model for a wide range of parameter values. Table 2 reports the results of a representative computer simulation.[18] As shown, the model has three steady state equilibria when $\alpha = 1$: two stable equilibria with extreme R&D behavior ($C = 0$ and $C = +\infty$) and one unstable equilibrium with positive rates of both innovation and imitation. As we lower α, these three equilibria become more similar until two of the three equilibria merge and disappear. Table 2 reveals that convergence occurs very quickly and for $\alpha \leq 0.96$, the model has only one steady state equilibrium. This equilibrium that survives has the same properties as were derived for the square-root R&D model. To summarize, computer simulations of the general model suggest that the properties derived in sections 3 and 4 (assuming that $\alpha = \frac{1}{2}$) hold for a wide range of α values and only fail to hold when α is very close to one.

7 Related Results

In this section, we review some of the models of innovation and imitation in the literature and discuss the relationships with the present paper.

There are a few recent articles that may appear, at first glance, to consider issues that are similar to those addressed in this paper. For example, Rustichini and Schmitz (1991) offer a general equilibrium model of innovation and imitation and examine the impact of subsidies on growth. Nevertheless, their approach differs from ours in a number of important ways. To begin with, R-S study a model of human capital accumulation by workers while we study a model of R&D investment by firms.[19] In their model, time spent "innovating" leads to immediate human capital accumulation whereas time spent "imitating" leads to delayed human capital accumulation. In our model, a firm innovates by developing a higher quality product and a firm imitates by copying another firm's product. Thus, in our framework, the whole point of engaging in either type of R&D is to acquire market power and associated economic profits. Whereas we carefully model how the R&D behavior of firms leads to changing market concentration levels over time, R-S simply assume that perfect

[18] All the equilibrium R&D intensities reported in Table 2 were obtained assuming that $\lambda = 1.5$, $A = .38$, $B = .472$, $\rho = .03$, $\mu = 1.1$, $L = .96$, $F = G = .025$, $s_I = s_C = 0$, $\phi = 1.2$ and $\sigma = 1.36$.

[19] Although R-S do not refer to their model as one of human capital accumulation, their assumption of perfect competition in all product markets and zero profits makes it impossible for their analysis to capture the dynamics of markets in which firms innovate and imitate in order to earn economic profits and capture market power. In fact, the structure of their model is very similar to that of Lucas' (1988) model of human capital accumulation. By comparing R-S's differential equation that defines the return to innovating with Lucas' differential equation that defines the reward to human capital accumulation, it is easy to see the similarity of the underlying mathematical structure.

competition prevails in all product markets and that firms earn zero economic profits.

Another seemingly related paper is by Jovanovic and MacDonald (1994). This paper offers an analysis of the evolution of a single industry in which firms engage in both innovative and imitative R&D. Our approach differs from theirs in a number of important ways. For example, while their analysis is partial equilibrium, we study a general equilibrium model of an entire economy. Furthermore, J-M use two strong assumptions which effectively rule out analysis of the issues that interest us. First, they assume that all firms are price takers. In other words, no matter how successful a firm is in reducing its production costs, it continues to have zero market share (and thus, there is no effect on market price). In our paper, the race to innovate, the race to imitate, and the product market competition between firms are all carefully modeled. Successful innovation results in a new market leader with substantial market power and successful imitation leads to greater product market competition and lower market prices. Second, J-M assume that there is an upper bound on technological progress. As a result, technological change eventually slows down as the industry "matures." The main question that we ask in this paper, "what is the effect of R&D subsidies on long-run economic growth?" has a trivial answer in the J-M model since economic growth does not occur in the long run.

More closely related to our paper is Grossman and Helpman (1991b), who study a North-South trade model where only firms in the South imitate and only firms in the North innovate. Firms in the North can also do imitative R&D but since imitation is assumed to involve no product differentiation (unlike in the present paper), it is not profitable for northern firms to imitate other northern firms' products. We view our paper as complementing G-H. We study the effects of R&D subsidies when firms in advanced countries copy other firms' ideas. G-H study the effects of R&D subsidies when all imitation occurs in the less developed South and is driven by lower southern wages. Their model is useful for understanding technology transfer to less developed countries whereas our model is more useful for understanding technology transfer inside advanced countries. Many innovations are copied by other firms in the North long before production shifts to the South. G-H find that R&D subsidies in the North promote growth, R&D subsidies in the South retard growth in the case of relatively efficient followers, and promote growth in the case of relatively inefficient followers. These results are closely related to Theorem 3. G-H do not obtain clean results about the effects of R&D subsidies on industry-level R&D intensities, comparable to Theorem 2. For example, they conclude that a higher imitative R&D subsidy may either increase or decrease the intensity of imitation targeted at any given Northern product. We show that a higher imitative R&D

subsidy unambiguously increases the imitative R&D intensity C. Perhaps the most surprising difference between the two papers, however, concerns the R&D technologies employed. G-H obtain plausible conclusions assuming constant returns to both innovative and imitative R&D. When we make the same assumptions, the only equilibrium with both innovation and imitation has perverse comparative steady state properties and is unstable (Theorem 5). Only when we assume substantial decreasing returns to R&D do we obtain reasonable sounding conclusions about the effects of R&D subsidies.

Most closely related to our paper is a recent paper by Cheng and Tao (1994), who also study a closed-economy model of innovation and imitation. They begin by showing that the steady state equilibrium in Segerstrom (1991) is unstable, a result comparable to Theorem 5.[20] However, to obtain a stable equilibrium with both innovation and imitation, C-T adopt a fundamentally different approach. They continue to assume that there are constant returns to R&D at the industry level but instead suppose that there are sufficiently strong *negative* R&D spillovers across industries. Firms becomes less productive in R&D when firms in other industries do more R&D. Due to different assumptions about product market competition, C-T find that the reward for innovating is higher in industries with two leaders than in industries with one leader. As a result, firms only do innovative R&D in industries with two leaders, and economic growth is positively related to both the innovation rate and the imitation rate in each industry. Higher innovative R&D subsidies may retard growth because imitation rates decrease and higher imitative R&D subsidies may stimulate growth because imitation rates increase. C-T conclude that the growth effects of both innovative and imitative R&D subsidies are ambiguous. In constrast, we find that firms do innovative R&D in all industries, innovative R&D subsidies stimulate growth and imitative R&D subsidies retard growth (Theorem 3).

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[20]Zeng (1995) develops a model of innovation and imitation with constant returns to R&D and then shows that R&D subsidies have normal steady-state effects. The key assumption driving Zeng's results is that all research is of a general unfocused nature. We reach opposite conclusions in section 6 because we assume (as do most papers in the Schumpeterian growth literature) that firms can focus their research efforts on finding improvements in particular industries.

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Appendix A

In this appendix, we establish various properties of the steady state conditions.

First, we examine the properties of the mutual R&D condition. From (13), it is clear that it goes through the origin in (C,I) space and has no strictly positive C-intercept or I-intercept. (13) can be directly rearranged to obtain a quadratic equation in I, namely, $f(I) \equiv c_0 + c_1 I + c_2 I^2 = 0$. Since $c_0 < 0$, and $c_2 > 0$ for all $C > 0$, this quadratic equation must have a strictly positive and a strictly negative root for all $C > 0$. Thus, in the positive orthant, the mutual R&D condition is a *function* of C for all $C \geq 0$. The mutual R&D condition can also be solved as a quadratic equation in C. Using the same reasoning, we are able to conclude that the mutual R&D condition is a *function* of I for all $I \geq 0$. Putting these two results together, we conclude that the mutual R&D condition is a globally upward sloping function in (C,I) space.

Next, consider the imitative R&D condition (14). It is straightforward to verify that it always has a unique strictly positive I-intercept and a unique strictly positive C-intercept. Multiplying both sides of the imitative R&D condition by $(I + C)$, for each fixed $I > 0$, we can solve for a quadratic equation in C, $f(C) \equiv c_0 + c_1 C + c_2 C^2 = 0$. c_2 is unambiguously positive and c_0 is negative if and only if $I < I$-intercept. Thus we are guaranteed that the quadratic equation has a unique strictly positive root for any $I < I$-intercept. For $I = I$-intercept, $c_0 = 0$, and $c_1 > 0$, implying that the quadratic equation has no strictly positive root. Since c_0, c_1, and c_2 are all increasing functions of I, for $I > I$-intercept, the quadratic equation has no strictly positive root. Thus, we conclude that in the positive orthant of (C,I) space, the imitative R&D condition is a *function* of I when $0 \leq I \leq I$-intercept, and for $I > I$-intercept, there exist no positive C-values which satisfy (14).

The imitative R&D condition can also be solved as a cubic equation in I, $f(I) \equiv c_0 + c_1 I + c_2 I^2 + c_3 I^3 = 0$. The coefficient c_3 is unambiguously positive and equal to a. The coefficient c_2 is unambiguously positive for all $C > 0$. The coefficient c_0 is unambiguously negative for all $C < C$-intercept and unambiguously positive for all $C > C$-intercept. Thus we are guaranteed a unique strictly positive root to the cubic equation in I for any $C < C$-intercept. For $C = C$-intercept, the cubic equation has a strictly positive root when $c_1 < 0$, which holds when $L > 0$ is sufficiently small. Thus, when the labor force is sufficiently small, the cubic equation has two strictly positive roots when C is slightly greater than the C-intercept. Since c_2, c_1, and c_0 are all increasing functions of C for all $C > C$-intercept, when the labor force is sufficiently large, the cubic equation has no strictly positive roots for any $C > C$-intercept. Putting all this information about the imitative R&D condition together, we have established that it is globally downward sloping in (C,I) space when the labor force is sufficiently large. Otherwise, when the labor force is relatively small, it is backward-bending, downward sloping for high I-values and upward sloping for low I-values.

By multiplying both sides of (15) by $(I + C)(\rho + I)[\frac{\sigma-1}{\sigma} + \frac{C}{\rho+I}\frac{\phi-1}{2\phi}]$, for each fixed $C > 0$, we can solve for a fourth degree polynomial equation in I, $f(I) \equiv c_0 + c_1 I + c_2 I^2 + c_3 I^3 + c_4 I^4 = 0$. The coefficients c_4 and c_3 are unambiguously positive for all $C > 0$, and the coefficient c_0 is unambiguously negative for all $C > 0$. Thus we are guaranteed at least one positive root to the fourth degree polynomial equation for all $C > 0$. To establish that there exists exactly one root, we need to examine more carefully the properties of $c_2(C)$ and $c_1(C)$. $c_2(C)$ is globally increasing in $C > 0$. $c_1(C)$ is a cubic polynomial in C and can be written as $c_1(C) \equiv d_0 + d_1 C + d_2 C^2 + d_3 C^3$. Since $d_3 > 0$, $d_2 > 0$, and $d_0 < 0$ unambiguously, the equation $c_1(C) = 0$ has exactly one strictly positive root and c_1 rises from negative to positive as C increases. We only have to worry about more than one root to the fourth degree polynomial equation if $c_1 > 0$ and $c_2 < 0$ for some $C > 0$. This possibility can be ruled out by showing that when $c_2 = 0$, $c_1 < 0$. Straightforward but tedious calculations reveal that this is indeed the case and the innovative R&D condition is a *function* of C for all $C \geq 0$.

Differentiation reveals that the left hand side of (15) is globally increasing in C for all fixed

$I > 0$, and globally increasing in I when $C = 0$. Thus, for any $I > I$-intercept, there exists no $C \geq 0$ which satisfies (15), and for any $I \leq I$-intercept, there exists at most one $C \geq 0$ which satisfies (15). We conclude that the innovative R&D condition must be globally downward sloping in (C,I) space.

Now consider (20) in the linear R&D model. By multiplying both sides of (20) by $(I + C)$, for each fixed $C > 0$, we can solve for a quadratic equation in I. Assumption A1 then guarantees that the quadratic equation has a strictly positive and a strictly negative root. Focusing in on the positive root, I is a *function* of C for all $C > 0$. Inspection also reveals that C is a *function* of I for all $I < I$-intercept. Thus the imitative R&D condition must be globally downward sloping in (C,I) space.

Finally, consider the innovative R&D condition. Differentiation reveals that the left hand side of (21) is globally increasing in C for all fixed $I > 0$, and globally increasing in I when $C = 0$. Thus C is a *function* of I for all $I \in (0, I$-intercept), when there is a C-intercept. If there is no C-intercept, then C is a *function* of I for all $I < I$-intercept where (21) can be satisfied. Furthermore, for any $I > I$-intercept, there exists no $C \geq 0$ which satisfies (21). Thus, if we can establish that I is also a *function* of C, we will be able to conclude that the innovative R&D condition is globally downward sloping.

By multiplying both sides of (21) by $(I + C)(\rho + I)[\frac{\sigma-1}{\sigma} + \frac{C}{\rho+I}\frac{\phi-1}{2\phi}]$, for each fixed $C > 0$, we can solve for a cubic equation in I, $f(I) \equiv c_0 + c_1 I + c_2 I^2 + c_3 I^3 = 0$. The coefficient c_3 is unambiguously positive. The coefficient c_0 is unambiguously negative for all $C < C$-intercept and unambiguously positive for all $C > C$-intercept. Thus we are guaranteed at least one positive root to the cubic equation in I for any $C < C$-intercept. Since c_2 is globally increasing in $C > 0$, $c_1(0) < 0$, and $c_1(C) = 0$ has exactly one positive root, we can guarantee that the cubic equation in I has exactly one positive root for any $C < C$-intercept, by showing that when $c_2 = 0$, $c_1 < 0$. Straightforward but tedious calculations reveal that this is indeed the case and the innovative R&D condition is globally downward sloping in (C,I) space.

Appendix B

In this appendix, we show that there exists an equilibrium transition path which converges to the interior steady state equilibrium in the linear R&D model.

First, we solve for the reward for imitating on an equilibrium transition path. Over a time interval dt, the shareholders of a successful imitator receive a dividend $\pi^C(t)\,dt$ and the value of the firm appreciates by $\dot{v}_C(t)\,dt = 0$ [since imitative R&D profit maximization implies that $v_C(t) = b(1 - s_C)$ at each point in time t]. Because the firm is targeted by other firms that conduct innovative R&D to discover the next higher quality product, the shareholders suffers a loss of $v_C(t)$ if further innovation occurs. This event occurs with probability $I(t)\,dt$. Efficiency in financial markets requires that the expected rate of return from holding a stock of the firm is equal to the riskless rate of return $r(t)\,dt$ that can be obtained through complete diversification: $\frac{\pi^C(t)}{v_C(t)}dt - \left[\frac{v_C(t)-0}{v_C(t)}\right] I(t)\,dt = r(t)\,dt$. Taking limits as dt approaches zero, yields

$$\frac{\frac{\phi-1}{2\phi}E(t)}{r(t)+I(t)} = b(1 - s_C), \qquad (B1)$$

which is the out-of-steady-state counterpart to (10) when $\alpha = 1$.

The reward for innovating on an equilibrium transition path can be similarly calculated. Over a time interval dt, the shareholders of a successful innovator receive a dividend $\pi^L(t)\,dt$ and the value of the firm appreciates by $\dot{v}_I(t)\,dt = 0$ [since innovative R&D profit maximization implies that $v_I(t) = a(1 - s_I)$ at each point in time t]. Because the firm is targeted by other R&D firms, the shareholders suffers a loss of $v_I(t)$ if further innovation occurs and a loss of $v_I(t) - v_C(t)$ if imitation occurs. Now efficiency in financial markets requires that $\frac{\pi^L(t)}{v_I(t)}dt - \left[\frac{v_I(t)-0}{v_I(t)}\right] I(t)\,dt - \left[\frac{v_I(t)-v_C(t)}{v_I(t)}\right] C(t)\,dt = r(t)\,dt$. Taking limits as dt approaches zero, yields

$$\frac{\frac{\sigma-1}{\sigma}E(t) + C(t)b(1-s_C)}{r(t)+I(t)+C(t)} = a(1 - s_I), \qquad (B2)$$

which is the out-of-steady-state counterpart to (9) when $\alpha = 1$.

The proportion of industries which have only one quality leader $\gamma(t)$ is a state variable which gradually adjusts over time. Its time path satisfies the differential equation

$$\dot{\gamma}(t) = [1 - \gamma(t)]I(t) - \gamma(t)C(t) \qquad (B3)$$

since γ increases when innovation occurs in $1-\gamma$ industries and γ decreases when imitation occurs in γ industries [note that $\dot{\gamma}(t) = 0$ implies (11)]. Outside of the steady state equilibrium, the full employment of labor condition (12) becomes

$$\frac{\gamma(t)}{\sigma}E(t) + \frac{1-\gamma(t)}{\phi}E(t) + \gamma(t)bC(t) + aI(t) = L \qquad (B4)$$

when $\alpha = 1$. In the linear R&D model, any equilibrium transition path with both innovation and imitation must satisfy (4), (B1), (B2), (B3) and (B4) for all time t.

Solving (B1) for $I(t)$, solving (B2) for $C(t)$ and then substituting into (B4), we obtain an interest rate function

$$r(\gamma, E) \equiv \frac{E}{a}\left\{\frac{\gamma}{\sigma} + \frac{1-\gamma}{\phi}\frac{a(\phi-1)}{2\phi b(1-s_C)} + \gamma b\Psi\right\} - \frac{L}{a} \qquad (B5)$$

where assumptions A1 and A2 guarantee that $\Psi \equiv \left[\frac{\sigma-1}{\sigma a(1-s_I)} - \frac{\phi-1}{2\phi b(1-s_C)}\right] / \left[1 - \frac{b(1-s_C)}{a(1-s_I)}\right]$ is strictly positive. Plugging the $I(t)$ and $C(t)$ expressions into (B3) using (B5) yields

$$\dot{\gamma} = (1-\gamma)\frac{L}{a} - \frac{E}{a}\left\{\gamma\Psi a + \frac{\gamma(1-\gamma)}{\sigma} + \frac{(1-\gamma)^2}{\phi} + (1-\gamma)\gamma b\Psi\right\}. \qquad (B6)$$

Taking into account (B3), (4) and (B6) represent a system of two nonlinear differential equations whose properties can be analyzed by drawing possible phase diagrams in (γ, E) space. Having already established that the linear R&D model has a unique interior steady state equalibrium, the $\dot{\gamma} = 0$ and $\dot{E} = 0$ curves must intersect exactly once in the strictly positive orthant of (γ, E) space. The $\dot{\gamma} = 0$ curve is a continuously differentiable function of γ and goes through the point $(0, 1)$. Since $\dot{\gamma}$ is decreasing in E, above this curve, $\dot{\gamma} < 0$. The $\dot{E} = 0$ curve is also a continuously differentiable function of γ and takes on a strictly positive value when $\gamma = 1$. Since $\dot{E}$ is increasing in E, above this curve, $\dot{E} > 0$. Using these properties, it is straightforward to verify that the interior steady state equilibrium is a saddlepoint equilibrium. The saddlepath is upward sloping if $\dot{E} = 0$ has a positive slope, is downward sloping if $\dot{E} = 0$ has a negative slope and coincides with $\dot{E} = 0$ if this function is a horizontal line. Jumping immediately onto this saddlepath and staying on it forever represents an equilibrium transition path for the linear R&D model. This transition path converges asymptotically to the interior steady state equilibrium.

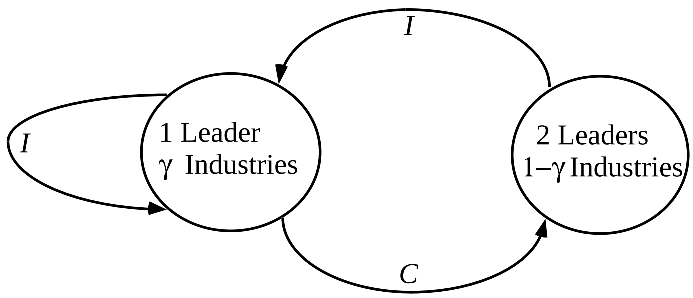

Figure 1: Steady State Pattern

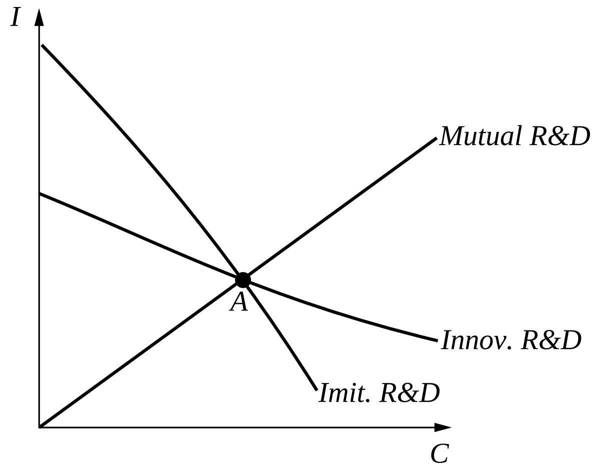

Figure 2: The “Large World” Interior Steady State Equilibrium

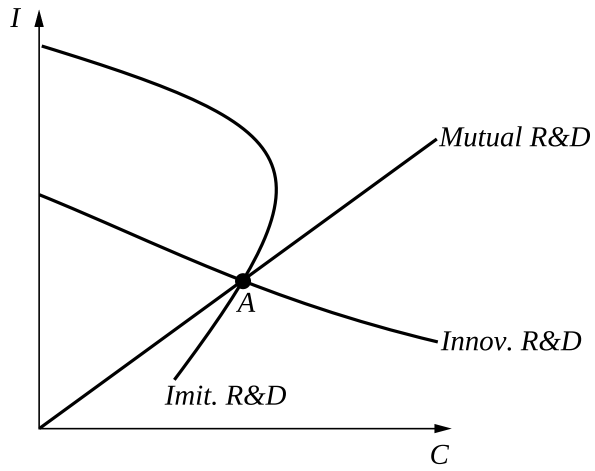

Figure 3: The “Small World” Interior Steady State Equilibrium

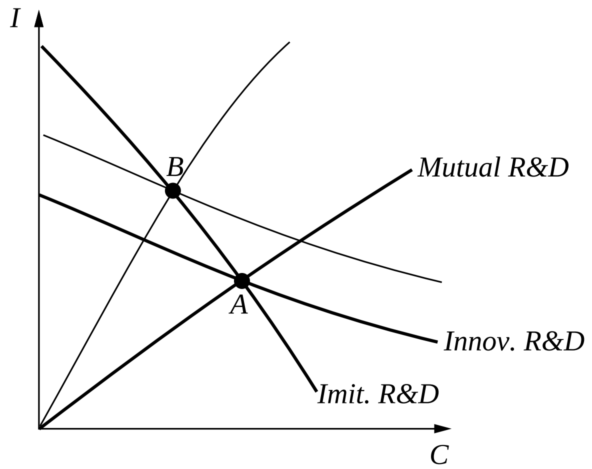

Figure 4: The Effect of an Innovative R&D Subsidy

Main Example						
s_I	s_C	B	I	C	g	U
0	0	3.49	.012	.333	.005	5.9
.821			.044	.153	.018	11.6
.761	-1.50		.043	.064	.018	11.8
.716	.716		.026	.772	.011	7.9
.639	.639	0	.045	0	.018	11.2

High-Growth Example						
s_I	s_C	B	I	C	g	U
0	0	4.85	.050	.333	.020	21.2
.897			.179	.066	.072	54.0
.871	$-\infty$		.179	0	.072	67.6
.894	.894		.127	.919	.052	39.6
.871	.871	0	.179	0	.072	67.6

Table 1: Policy Simulation Results

	Equilibrium 1		Equilibrium 2		Equilibrium 3	
α	I	C	I	C	I	C
1.0	.0735	0	.0123	.337	.00517	$+\infty$
.99	.0708	3.7x10^{-42}	.0106	.401	.00498	1.3x10^{7}
.98	.0682	8.8x10^{-22}	.0088	.534	.00490	246.7
.97	.0657	5.3x10^{-15}	.0064	1.217	.00531	3.925
.96	.0633	1.2x10^{-11}				
.90	.0506	1.3x10^{-5}				
.70	.0229	.0035				
.50	.0121	.0064				

Table 2: Multiple Equilibria Simulation Results

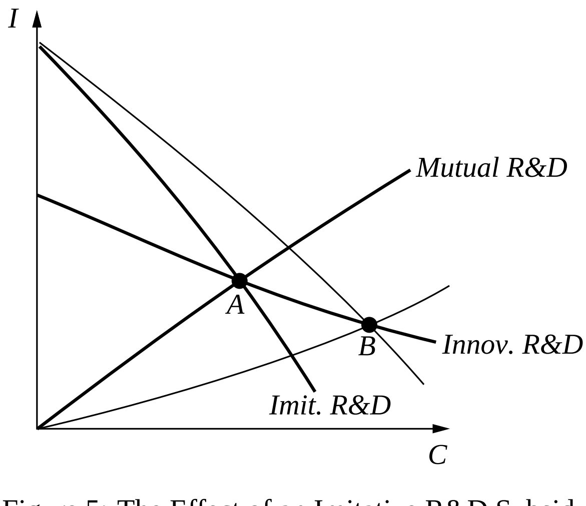

Figure 5: The Effect of an Imitative R&D Subsidy

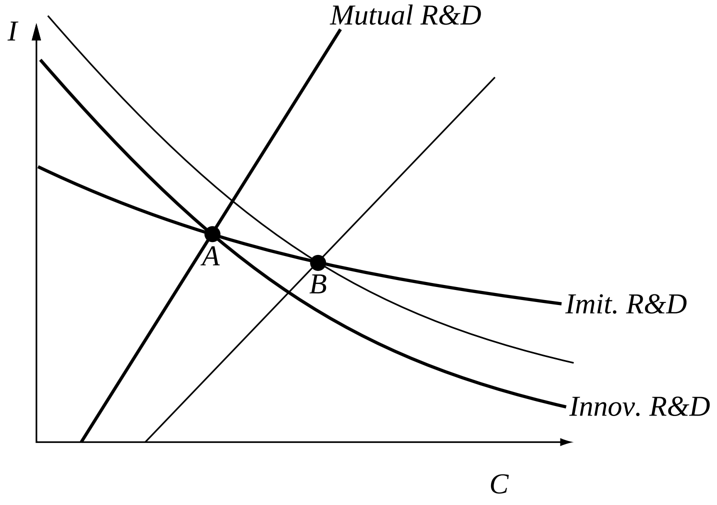

Figure 6: The Effect of an Innovative R&D Subsidy in the Linear R&D Model

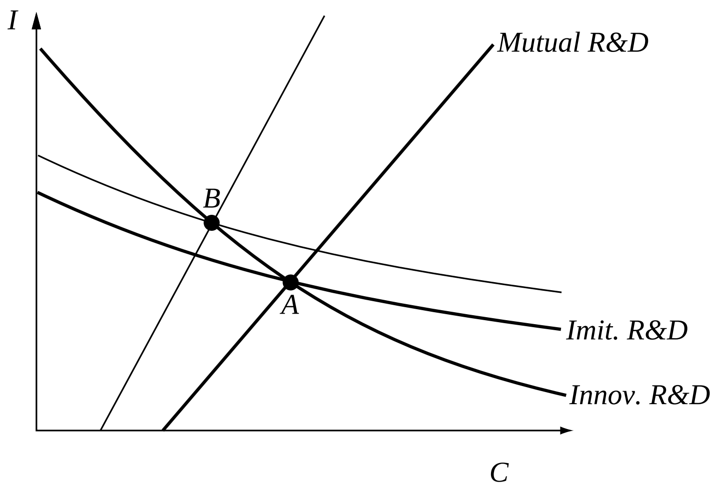

Figure 7: The Effect of an Imitative R&D Subsidy in the Linear R&D Model

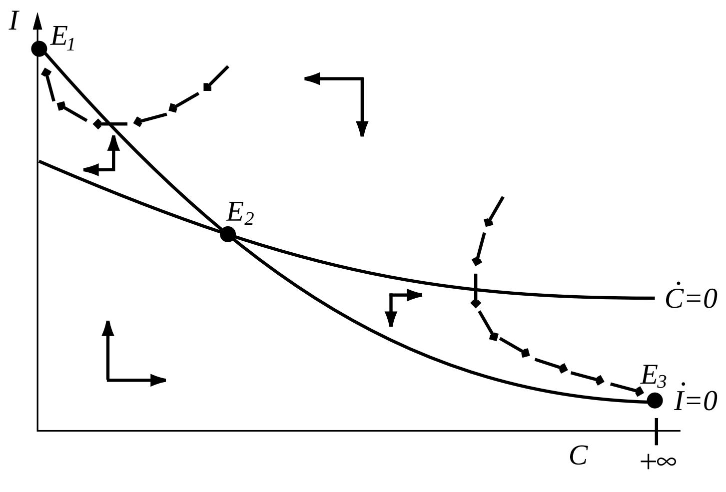

Figure 8: A Phase Diagram for the Linear R&D Model