The Long-Run Growth Effects of R&D Subsidies

Paul S. Segerstrom
Stockholm School of Economics

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Abstract: This paper presents a generalized version of Howitt's (1999) model of R&D-driven growth without scale effects and a complete characterization of the long-run growth effects of R&D subsidies. R&D subsidies can either promote or retard long-run economic growth and surprisingly, the growth-retarding outcome occurs for a wide range of plausible parameter values. This paper also presents a new intuitive explanation for why R&D subsidies can have long-run growth effects (both positive and negative).

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Author mailing address: Professor Paul S. Segerstrom, Stockholm School of Economics, Box 6501, SE-113 83 Stockholm, Sweden.

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1 Introduction

Perhaps the main conclusion that emerges from the R&D-driven endogenous growth literature is that public policies can have long-run growth effects by influencing the incentives firms have to innovate. Romer (1990), Segerstrom et al. (1990), Grossman and Helpman (1991) and Aghion and Howitt (1992) all find that R&D subsidies encourage firms to devote more resources to R&D activities and as a result increase the long-run rate of economic growth. Furthermore, because R&D subsidies promote growth, many other public policies that indirectly affect the R&D incentives of firms can also have long-run growth effects. For example, Rivera-Batiz and Romer (1991) show that lower tariffs between countries lead to a permanent increase in each country's rate of economic growth.

This literature has been challenged by Jones (1995), who points out that all of the above-mentioned endogenous growth models have an undesirable "scale effect" property, namely, that economic growth is faster when firms devote more resources to R&D. Since 1950, the number of scientist and engineers engaged in R&D in advanced countries has increased dramatically without generating any upward trend in economic growth rates. Furthermore, when Jones modifies Romer's (1990) model of horizontal innovation to eliminate the scale effect, he finds that doing so also eliminates the long-run growth effects of R&D subsidies.[1] In the Jones (1995) model, higher R&D subsidies increase the relative size of the R&D sector, but have no effect on the long-run rate of economic growth, which only depends on the population growth rate and other exogenous parameters. Segerstrom (1998) shows that the same results hold in a model of vertical innovation.[2]

Recently, Howitt (1999) has come to the defense of R&D-driven endogenous growth theory. He presents a model where firms can engage in both horizontal and vertical R&D activities. Like in Jones (1995) and Segerstrom (1998), Howitt's model does not have

[1] Firms engage in horizontal R&D to increase the number of industries in the economy (create entirely new products). Firms engage in vertical R&D to improve the quality of existing products.

[2] In Romer's (1990) model of horizontal innovation, as the stock of knowledge in the economy increases over time, the productivity of researchers increases proportionately. To rule out scale effects, Jones (1995) assumes that new knowledge contributes to the productivity of researchers at a decreasing (instead of constant) rate. In Grossman and Helpman (1991), the productivity of researchers does not change over time. To rule out scale effects, Segerstrom (1998) modifies this model by assuming that innovating becomes progressively more difficult over time.

the scale effect property: the long-run economic growth rate is an increasing function of the population growth rate (instead of population level). However, all of the other forces determining the long-run economic growth rate are exactly the same as in the early R&D-driven endogenous growth models; in particular, R&D subsidies promote long-run economic growth. The Howitt model's steady state is broadly consistent with the evidence that Jones has argued contradicts R&D-driven endogenous growth theory.

Howitt's (1999) model should be viewed as a counterexample to Jones's claim that certain facts cannot be squared with R&D-driven endogenous growth models. This paper explores what happens more generally in these models. Howitt (1999) makes special assumptions about how the returns to horizontal and vertical R&D activities differ, and also about how the returns to both R&D activities change over time. In this paper, these assumptions are relaxed and a generalized version of Howitt's model is analyzed. Simple parameter conditions are derived which completely characterize when R&D subsidies promote long-run economic growth and when R&D subsidies retard long-run economic growth. Surprisingly, for a wide range of plausible parameter values, permanently higher R&D subsidies lead to a lower long-run rate of economic growth. This paper also presents a new intuitive explanation for why R&D subsidies can have long-run growth effects (both positive and negative). Even in the "normal" case where R&D subsidies promote growth, the reasons why are fundamentally different from those discussed in the early R&D-driven endogenous growth literature.[3]

The rest of this paper is organized as follows: The model is presented in section 2 and the effects of R&D subsidies (both general and targeted) are studied in section 3. The new intuitive explanation for why R&D subsidies sometimes promote growth and sometimes retard growth is also developed in section 3. Section 4 discusses the related work by Li (1999) on two sector R&D-driven growth models. Finally, section 5 summarizes the conclusions reached in the paper.

[3]In this paper, R&D subsidies having negative long-run growth effects is not a "Giffen good phenomenon", a theoretical outcome that only occurs under extreme circumstances. Rather, this outcome occurs roughly 50% of the time (for example, if horizontal R&D subsidies promote growth, then vertical R&D subsidies retard growth).

2 The Model

The model presented in this section is the same as in Howitt (1999) but with more general assumptions about R&D and a constant (instead of decreasing) returns to scale production technology.

2.1 Production

Consumption goods and R&D services are both produced by firms under conditions of perfect competition using the same constant returns to scale production function. The inputs in the production process are labor and a continuum of intermediate products. Specifically, the total output of the economy at any date t is

$$Y_t = C_t + H_t + V_t = L_{yt}^{1-\alpha} \int_0^{N_t} A_{it} x_{it}^{\alpha}\, di \tag{1}$$

where Y_t is gross output, C_t is consumption, H_t is horizontal R&D expenditure, V_t is vertical R&D expenditure, L_{yt} is the labor devoted to producing output, N_t is the measure of how many different intermediate products exist at time t, x_{it} is the flow of intermediate product i used throughout the economy, A_{it} is a productivity parameter attached to the latest version of intermediate product i, and $\alpha \in (0, 1)$ is a parameter which determines the elasticity of demand for intermediate products. Each intermediate product is in turn produced using labor only, according to the production function $x_{it} = L_{it}$, where L_{it} is the input of labor in industry i.

Howitt (1999) assumes that the final goods production function is $Y_t = \int_0^{N_t} A_{it} x_{it}^{\alpha}\, di$. The main results in this paper go through without modification using Howitt's decreasing returns to scale production function. However, the model then has an unattractive implication: for a wide range of parameter values, the steady-state rate of economic growth is negative even though there is a positive steady-state rate of technological change. This occurs because the positive steady-state rate of technological change is more than offset by decreasing returns to scale in production. By assuming constant returns to scale in production, this outcome is avoided and the steady-state rate of economic growth is always positive.

Consider a typical firm j that produces final goods, that is, either consumption goods or R&D services. At time t, this firm solves the profit maximization problem

$$\max_{L_{yjt},\,x_{ijt}} L_{yjt}^{1-\alpha}\int_0^{N_t} A_{it}x_{ijt}^{\alpha}\,di - \int_0^{N_t} p_{it}x_{ijt}\,di - w_tL_{yjt}$$

where L_{yjt} is the labor employed by firm j to produce final output, x_{ijt} is the flow of intermediate input i used by firm j, p_{it} is the price of intermediate input i and w_t is the wage rate for labor, measured in units of final output (the numeraire for all prices). Solving for profit maximizing behavior yields the first order condition $p_{it} = A_{it}\alpha(L_{yjt}/x_{ijt})^{1-\alpha}$. However, since all firms face the same prices p_{it}, all firms must choose the same input ratios ($x_{ijt}/L_{yjt} = x_{it}/L_{yt}$ for all j) and this first order condition can be written more simply as

$$p_{it} = A_{it}\alpha(L_{yt}/x_{it})^{1-\alpha}. \tag{2}$$

This is the inverse demand function for intermediate input i by the producers of final goods and it implies that the elasticity of demand for each intermediate input is $\frac{-1}{1-\alpha}$. The second first order condition for profit maximization is

$$w_t = (1-\alpha)\int_0^{N_t} A_{it}\left(\frac{x_{it}}{L_{yt}}\right)^{\alpha} di, \tag{3}$$

which helps pin down the equilibrium real wage rate for labor at time t. It is easily verified that when (2) and (3) hold, the firms that produce final goods all earn zero economic profits.

2.2 Consumers

Each consumer lives forever, has linear additive preferences over consumption at each point in time and a constant rate of time preference $\rho > 0$. Thus, a consumer born at time t_0 maximizes the discounted utility function

$$\int_{t_0}^{\infty} e^{-\rho t}c(t)\,dt$$

subject to the usual intertemporal budget constraint, where $c(t)$ is the consumer's expenditure at time t. Then the market interest rate must equal ρ throughout time.

2.3 Innovation

Firms engage in vertical R&D activities with the goal of developing higher quality intermediate products. Each vertical innovation is associated with a higher value of A_{it} for some industry i. Let $A_t \equiv \max\{A_{it}; i \in [0, N_t]\}$ denote the leading-edge productivity parameter at time t. A vertical innovation at time t in industry $i \in [0, N_t]$ results in a new intermediate product which embodies the leading-edge productivity parameter A_t. The Poisson arrival rate of vertical innovations in each industry $i \in [0, N_t]$ at time t is denoted by ϕ_t. This Poisson arrival rate does not vary across industries at time t because the reward for innovating is the same in all industries (is proportional to A_t, as will be established later) and R&D costs are also assumed to be the same across industries.

As in Caballero and Jaffe (1993) and Howitt (1999), the leading-edge productivity parameter A_t grows over time as a result of knowledge spillovers produced by vertical innovations:

$$g_{At} \equiv \frac{\dot{A}_t}{A_t} = \left(\frac{\sigma}{N_t}\right)(\phi_t N_t) = \sigma\phi_t, \tag{4}$$

where $\sigma > 0$ is a given R&D spillover parameter. Equation (4) has a natural interpretation. The size of the knowledge spillovers is proportional to the aggregate flow of vertical innovations $\phi_t N_t$ in the economy. The factor of proportionality is given by σ/N_t and can be interpreted as the marginal impact of each vertical innovation on the stock of public knowledge which researchers use. Division by N_t in the factor of proportionality captures the idea that as the economy develops an increasing number of intermediate products, each vertical innovation has a smaller impact on the aggregate economy.

Firms engage in horizontal R&D activities with the goal of developing different intermediate products, that is, creating entirely new industries. Horizontal innovation is associated with increases in N_t over time. At time t, each horizontal innovation results in a new intermediate product i whose productivity parameter A_{it} is drawn randomly from the existing distribution of productivity parameters across industries.

2.4 Intermediate Product Markets

Any firm that innovates immediately receives a patent on its innovation and patent rights are strictly enforced. Thus, a firm that innovates does not have to worry about other firms copying its product.[4] Given the absence of copying, a firm that horizontally innovates does not have to deal with competitors in its newly created industry and thus, this firm can earn positive profit flows until the next vertical innovation in its industry occurs. A firm that vertically innovates enters into Bertrand price competition with the previous incumbent in its industry, a firm that produces a lower quality product. It is either in the interest of the new industry leader to practice limit pricing (as in Grossman and Helpman (1991)) or to charge an unconstrained monopoly price, depending on whether the quality difference between the two competing firms is small or large. In either case, the previous incumbent does not ever sell any output or earn any profits in equilibrium. Faced with these grim future prospects, the previous incumbent immediately exits and then cannot threaten to re-enter the industry. Thus, a firm that vertically innovates also earns monopoly profits until the next vertical innovation in its industry occurs.[5]

The incumbent monopolist of intermediate product i has total cost of production $w_t x_{it}$ and the (inverse) demand for its product is given by (2). Solving this firm's profit maximization problem $\max_{x_{it}} \pi_{it} = (p_{it} - w_t)x_{it}$ yields the standard monopoly markup over marginal cost $p_{it} = w_t/\alpha$, the quantity of intermediate product i that is supplied:

$$x_{it} = L_{yt} \left(\frac{A_{it}\alpha^2}{w_t} \right)^{1/(1-\alpha)}, \qquad (5)$$

and the monopoly profit flow in industry i:

$$\pi_{it} = L_{yt}\alpha(1-\alpha)A_{it} \left(\frac{A_{it}\alpha^2}{w_t} \right)^{\alpha/(1-\alpha)}. \qquad (6)$$

Equation (6) implies that the profits earned by an innovative firm can either rise or fall over time. On the one hand, population growth (operating through increases in L_{yt}) leads to

[4] For a R&D-driven endogenous growth model where patent protection is not perfect and firms copy products developed by other firms, see Davidson and Segerstrom (1998).

[5] As in Grossman and Helpman (1991) and Aghion and Howitt (1992), since current industry leaders have less to gain from vertically innovating than other firms, they do not participate in vertical R&D races and vertical innovation always results in the previous leader firm being driven out of business. For models where industry leaders have R&D cost advantages and as a result participate in vertical R&D races, see Segerstrom and Zolnierek (1999) and Segerstrom (2000).

increases in the demand for the innovative firm's product over time, and these demand increases contribute to increasing the innovative firm's profits over time. On the other hand, increases in the real wage w_t associated with economic growth mean that the firm's production costs increase over time and these production cost increases contribute to decreasing the innovative firm's profits over time.

2.5 The Rewards for Innovating

The reward for a vertical innovation at time t is the expected discounted value of profit flows earned by the innovative firm before being replaced by the next innovator in its industry, which can be expressed as:

$$\Pi_{vt} = \int_t^\infty e^{-\int_t^\tau (\rho+\phi_s)ds} \hat{\pi}_{t\tau} d\tau, \tag{7}$$

where $\hat{\pi}_{t\tau}$ is the monopoly profit flow at time τ for a firm whose technology is of vintage t. In (7), the instantaneous discount rate applied to the profits earned by an innovative firm is the market interest rate ρ plus the rate of creative destruction ϕ_s; the latter being the instantaneous probability of further innovation in the industry under consideration. As was claimed earlier, the reward for a vertical innovation does not vary across industries at time t.

Since each horizontal innovation results in a new intermediate product whose productivity parameter is drawn randomly from the distribution of existing intermediate products, it follows from (6) and (7) that the expected value of a horizontal innovation is

$$\Pi_{ht} = E\left[(A_{it}/A_t)^{1/(1-\alpha)}\right] \Pi_{vt}. \tag{8}$$

As is shown in the Appendix, because the distribution of productivity parameters among new products at any time t is identical to the distribution across existing products at that time t, the distribution of relative productivity parameters $a_{it} \equiv A_{it}/A_t$ converges monotonically to the invariant distribution

$$\Pr(a_{it} \leq a) = F(a) \equiv a^{1/\sigma}; 0 < a \leq 1. \tag{9}$$

Since the focus in this paper is on the steady-state equilibrium properties of the model, the distribution of relative productivities is assumed to be $F(\cdot)$ at time $t = 0$, so the distribution of relative productivities does not change over time. It follows that

$$E\left[(A_{it}/A_t)^{1/(1-\alpha)}\right] = \int_0^1 a^{1/(1-\alpha)} F'(a)da = \Gamma^{-1} \tag{10}$$

where $\Gamma \equiv \frac{\sigma}{1-\alpha} + 1$.

2.6 Real Wage and Output Dynamics

Each incumbent monopolist charges the standard monopoly markup over marginal cost $p_{it} = w_t/\alpha$. Taking into account (2), this implies that all final good producers choose the same input ratio $x_{it}/L_{yt} = (A_{it}\alpha^2/w_t)^{1/(1-\alpha)}$. Substituting this equation and $\int_0^{N_t} A_{it}^{1/(1-\alpha)} di = A_t^{1/(1-\alpha)} N_t \int_0^1 a^{1/(1-\alpha)} F'(a)\, da = A_t^{1/(1-\alpha)} N_t \Gamma^{-1}$ into (3) yields the real wage dynamics condition

$$w_t = \frac{(1-\alpha)^{1-\alpha}\alpha^{2\alpha}}{\Gamma^{1-\alpha}} A_t N_t^{1-\alpha} \tag{11}$$

The real wage w_t rises over time as vertical R&D increases the leading-edge productivity parameter A_t and horizontal R&D increases the measure of industries N_t in the economy. Likewise, substituting into (1) using (5) and (11) yields the output dynamics condition

$$Y_t = \frac{\alpha^{2\alpha}}{(1-\alpha)^{\alpha}\Gamma^{1-\alpha}} L_{yt} A_t N_t^{1-\alpha}. \tag{12}$$

2.7 The Labor Market

The total supply of labor L_t is fixed inelastically at each time t by the population, which grows over time at the constant exogenous rate $g_L > 0$, that is, $L_t = L_0 e^{g_L t}$. Workers are either employed producing intermediate products or final goods. Thus, the full employment of labor condition is

$$\int_0^{N_t} x_{it}\, di + L_{yt} = L_t.$$

Substituting into this expression using (5) and (11) yields

$$L_{yt} = \frac{L_t}{1 + \frac{\alpha^2}{1-\alpha}}, \tag{13}$$

which pins down the employment of labor in the final goods sector throughout time. Equations (11), (12) and (13) together imply that the real wage rate of labor (w_t) grows over time at the same rate as GDP per worker (Y_t/L_t).

2.8 Vertical R&D

The Poisson arrival rate ϕ_{ijt} of vertical innovations in industry i by firm j at time t is given by

$$\phi_{ijt} = \frac{\lambda_v (V_{ijt})^{\delta} (K_{ijt})^{1-\delta}}{(A_t)^d}$$

where $\lambda_v > 0$ is a vertical R&D productivity parameter, V_{ijt} is firm j's vertical R&D expenditure flow, K_{ijt} is the firm-specific knowledge possessed by firm j that is useful for vertical R&D, the exponent $\delta < 1$ measures the degree of diminishing returns to vertical R&D expenditure, and the exponent $d > 0$ determines the rate at which research problems become more complex and harder to solve as the leading-edge productivity parameter A_t increases over time.

At each point in time t, the profit-maximizing vertical R&D firm j in industry i solves the problem $\max_{V_{ijt}} \ \phi_{ijt}\Pi_{vt} - V_{ijt}(1 - s_v)$, where s_v is the vertical R&D subsidy rate. The first order condition for this problem is

$$\frac{\delta \, \lambda_v \Pi_{vt}}{(A_t)^d} \left(\frac{V_{ijt}}{K_{ijt}} \right)^{\delta - 1} = 1 - s_v, \tag{14}$$

which is the usual requirement that the marginal expected benefit of an extra unit of vertical R&D equals its marginal cost. When $\delta < 1$, (14) implies that an increase in the reward for innovating Π_{vt} induces each firm j to increase its R&D effort V_{ijt}. Equation (14) also implies that $V_{ijt}/K_{ijt} = V_{it}/K_{it}$ for all j where $V_{it} \equiv \sum_j V_{ijt}$ and $K_{it} \equiv \sum_j K_{ijt}$, that is, each firm devotes resources to vertical R&D is proportion to the firm-specific knowledge that it possesses. I assume that there is symmetry across R&D firms (K_{ijt} is the same for all j) and that vertical R&D races are perfectly competitive (K_{ijt} is infinitesimally small). The latter assumption implies that the likelihood of any one firm winning a vertical R&D race is negligable.

As the economy grows over time and the stock of knowledge increases, researchers have more ideas to work with in developing new ideas, which by itself makes them more productive. I capture this basic insight in Romer (1990) by assuming that $K_{it} \equiv \sum_j K_{ijt} = Y_t/N_t$ for all i, that is, the total amount of firm-specific knowledge in each industry equals per industry output, which grows over time in equilibrium. Then the first order condition for vertical R&D profit maximization (14) can be written more simply (using $V_{it} = V_t/N_t$) as

$$\frac{\delta\,\lambda_v \Pi_{vt}}{(A_t)^d}(v_t)^{\delta-1} = 1 - s_v, \tag{15}$$

where $v_t \equiv V_t/Y_t$ is the fraction of GDP that is allocated to vertical R&D.

The returns to engaging in vertical R&D are assumed to be independently distributed across firms and over time. Thus, it is easily verified that the Poisson arrival rate of vertical innovations in each industry is

$$\phi_t = \sum_j \phi_{ijt} = \frac{\lambda_v (V_t/N_t)^{\delta}(Y_t/N_t)^{1-\delta}}{(A_t)^d} = \lambda_v v_t^{\delta} y_t \tag{16}$$

where $y_t \equiv Y_t/(N_t A_t^d)$. The Poisson arrival rate of vertical innovations ϕ_t is an increasing function of per industry vertical R&D expenditure V_t/N_t and the knowledge spillover term Y_t/N_t, both of which grow over time. Counterbalancing this, ϕ_t is a decreasing function of the R&D difficulty term A_t^d, which also grows over time.

2.9 Horizontal R&D

The discovery rate of new industries by firm j at time t is given by

$$\dot{N}_{jt} = \frac{\lambda_h (H_{jt})^{\gamma}(\mathcal{K}_{jt})^{1-\gamma}}{(A_t)^d}$$

where $\lambda_h > 0$ is a horizontal R&D productivity parameter, H_{jt} is firm j's horizontal R&D expenditure flow, $\mathcal{K}_{jt}$ is the firm-specific knowledge possessed by firm j that is useful for horizontal R&D, the exponent $\gamma < 1$ measures the degree of diminishing returns to horizontal R&D expenditure and as with vertical R&D, the exponent $d > 0$ determines the rate at which research problems become harder to solve as the leading-edge productivity parameter A_t increases over time.

At each point in time t, the profit-maximizing horizontal R&D firm j solves the problem $\max_{H_{jt}}\ \dot{N}_{jt}\Pi_{ht} - H_{jt}(1 - s_h)$ where s_h is the horizontal R&D subsidy rate. The first order condition for this problem is

$$\frac{\gamma\,\lambda_h \Pi_{ht}}{(A_t)^d}\left(\frac{H_{jt}}{\mathcal{K}_{jt}}\right)^{\gamma-1} = 1 - s_h, \tag{17}$$

that is, the marginal expected benefit of an extra unit of horizontal R&D equals its marginal cost. Equation (17) implies that $H_{jt}/\mathcal{K}_{jt} = H_t/\mathcal{K}_t$ for all j where $H_t \equiv \sum_j H_{jt}$ and $\mathcal{K}_t \equiv \sum_j \mathcal{K}_{jt}$, that is, each firm devotes resources to horizontal R&D is proportion to the firm-specific knowledge that it possesses. I assume that $\mathcal{K}_t \equiv \sum_j \mathcal{K}_{jt} = Y_t$, so the total stock of firm-specific knowledge useful for horizontal R&D grows over time at the same rate as the economy's gross output. Then the first order condition for horizontal R&D profit maximization (17) can be written more simply as

$$\frac{\gamma\,\lambda_h \Pi_{ht}}{(A_t)^d}(h_t)^{\gamma-1} = 1 - s_h, \tag{18}$$

where $h_t \equiv H_t/Y_t$ is the proportion of gross output devoted to horizontal R&D.

The growth rate of the measure of industries can now be determined by summing up the discovery rates for all the individual firms that engage in horizontal R&D:

$$g_{Nt} \equiv \frac{\dot{N}_t}{N_t} = \sum_j \frac{\dot{N}_{jt}}{N_t} = \frac{\lambda_h (H_t/N_t)^\gamma (Y_t/N_t)^{1-\gamma}}{(A_t)^d} = \lambda_h h_t^\gamma y_t \tag{19}$$

The rate at which the measure of industries grows over time g_{Nt} is an increasing function of per industry horizontal R&D expenditure H_t/N_t and the knowledge spillover term Y_t/N_t, both of which grow over time. Counterbalancing this, g_{Nt} is a decreasing function of the R&D difficulty term A_t^d, which also grows over time. This completes the description of the model.

Whereas Howitt (1999) assumes that there are constant returns to vertical R&D ($\delta = 1$) and diminishing returns to horizontal R&D ($\gamma < 1$), I allow for arbitrary degrees of diminishing returns to both R&D activities ($\delta, \gamma < 1$), motivated by the empirical evidence of significant diminishing returns to R&D reported in Kortum (1993) and Thompson (1996). Given the lack of clear evidence on how the returns to horizontal and vertical R&D activities differ, it is important to consider both possibilities: greater diminishing returns to horizontal R&D ($\delta > \gamma$) and greater diminishing returns to vertical R&D ($\delta < \gamma$).

Howitt (1999) also makes a special assumption about how the returns to R&D expenditure change over time ($d = 1$). This assumption implies that the patents-per-researcher ratio is constant over time when there are decreasing returns to scale in production (as Howitt assumed) and increases over time when there are constant returns to scale in production (as is assumed in this paper). In fact, the patents-per-researcher ratio has declined significantly over time in many countries (see Table 1) and to account for this trend, I allow for more

Table 1: Average Annual Growth Rate in the Patents-Per-Researcher Ratio

Country	Time Period	Growth Rate
United States	1965-1993	-2.18%
France	1965-1993	-6.07%
Japan	1965-1993	-0.11%
Sweden	1971-1993	-6.26%
United Kingdom	1969-1993	-5.74%

rapid growth in R&D difficulty as products become more complex ($d > 1$). The model's implications for the patents-per-researcher ratio are derived in the next section.[6]

3 Balanced Growth Properties

In this section, the balanced growth equilibrium properties of the model are analyzed. After establishing that the model has a unique balanced growth equilibrium where all endogenous variables grow at constant (not necessarily identical) rates over time, the circumstances under which R&D subsidies promote growth and retard growth are characterized. Intuitive explanations are provided for why both outcomes occur. The effects of subsidizing horizontal and vertical R&D at the same rate $s = s_h = s_v$ are studied first, followed by the more complicated case where horizontal and vertical R&D are subsidized at different rates.

Equation (1) implies that in any balanced growth equilibrium, both the fraction of GDP allocated to horizontal R&D and the fraction of GDP allocated to vertical R&D must be constant over time ($h_t = h$ and $v_t = v$ for all t). Since g_{At} and g_{Nt} must both be constant over time in a balanced growth equilibrium by definition, (4) implies that the Poisson arrival rate of vertical innovations must be constant over time also ($\phi_t = \phi$ for all t). It then follows from (16) that y_t must also be constant over time in any balanced growth equilibrium ($y_t =$

[6] In calculating the patents-per-researcher ratio, the rate of patenting is measured by the annual total number of patents granted to residents in WIPO (1983) and WIPO (various issues), and R&D employment is measured by the total number of R&D scientists and engineers in National Science Board (1998).

y for all t). Thus, the quality and variety growth rates can be written more simply as:

$$g_A = \sigma \lambda_v v^{\delta} y, \tag{20}$$

and

$$g_N = \lambda_h h^{\gamma} y. \tag{21}$$

3.1 Patenting Behavior

In a balanced growth equilibrium, the rate of patenting for horizontal innovations is $\dot{N}_t$ and the labor devoted to horizontal R&D is hL_{yt}. It follows from (12), (19) and the definition of y that the patents-per-researcher ratio associated with horizontal R&D at time t is given by

$$\frac{\dot{N}_t}{hL_{yt}} = \frac{\lambda_h \alpha^{2\alpha}}{(1-\alpha)^{\alpha}\Gamma^{1-\alpha}} \left(\frac{N_t^{1-\alpha}}{A_t^{d-1}}\right) h^{\gamma-1}.$$

The rate of patenting for vertical innovations is ϕN_t and the labor devoted to vertical R&D is vL_{yt}. It follows from (12), (16) and the definition of y that the patents-per-researcher ratio associated with vertical R&D at time t is given by

$$\frac{\phi N_t}{vL_{yt}} = \frac{\lambda_v \alpha^{2\alpha}}{(1-\alpha)^{\alpha}\Gamma^{1-\alpha}} \left(\frac{N_t^{1-\alpha}}{A_t^{d-1}}\right) v^{\delta-1}.$$

Both patents-per-researcher ratios decline over time if and only if $(d-1)g_A > (1-\alpha)g_N$. Thus $d > 1$ is a necessary condition for the overall patents-per-researcher ratio to decline over time. Given the evidence in Table 1, I assume that $d > 1$ holds in the remainder of the paper, since the patents-per-researcher ratio necessarily increases over time when $d \leq 1$.

3.2 Economic Growth

Let g denote the growth rate of the real wage w_t, that is, the economic growth rate for the economy. Differentiating (11) with respect to time yields

$$g = g_A + (1-\alpha)g_N. \tag{22}$$

In a balanced growth equilibrium, economic growth depends both on growth in the measure of industries g_N and on growth in the productivity of industries g_A.

3.3 The Population Growth Condition

In any balanced growth equilibrium, y_t must be constant over time. This has a strong implication. Substituting into the definition of y using (12) and (13), it follows that

$$y \equiv \frac{Y_t}{N_t A_t^d} = \left(\frac{L_t}{1 + \frac{\alpha^2}{1-\alpha}} \right) \left(\frac{\alpha^{2\alpha} A_t^{1-d} N_t^{-\alpha}}{\Gamma^{1-\alpha}(1-\alpha)^\alpha} \right).$$

Taking logs of both sides and then differentiating respect to time yields the population growth condition:

$$g_L = (d-1)g_A + \alpha g_N. \tag{23}$$

The rates at which both the leading-edge productivity parameter A_t and the measure of industries N_t can grow over time in a balanced growth equilibrium is constrained by the growth rate of the labor force g_L. With the productivity of researchers falling over time as the problems they face become more complex and harder to solve, the resources devoted to R&D must increase over time just to maintain a constant rate of economic growth. The growth rate of the labor force determines the rate at which the resources devoted to both horizontal and vertical R&D can increase over time in a balanced growth equilibrium and thus the growth rate of the labor force also helps determine how fast the economy can grow over time.

3.4 The Vertical R&D Condition

From (6), the monopoly profit flow at time τ for a firm whose technology is of vintage t is

$$\hat{\pi}_{t\tau} = L_{y\tau}\alpha(1-\alpha)A_t \left(A_t \alpha^2 / w_\tau \right)^{\alpha/(1-\alpha)}.$$

In a balanced growth equilibrium, (11) implies that $w_\tau = w_t e^{g(\tau - t)}$ and (13) implies that $L_{y\tau} = L_{yt} e^{g_L(\tau - t)}$. Thus evaluating the integral in (7) using (11) yields the expected discounted reward for a vertical innovation at time t

$$\Pi_{vt} = \frac{\alpha^{1+2\alpha}(1-\alpha)^{1-\alpha}\Gamma^\alpha L_{yt} A_t N_t^{-\alpha}}{\rho - g_L + \frac{g_A}{\sigma} + \frac{g\alpha}{1-\alpha}}.$$

Substituting this reward Π_{vt} back into (15) and simplifying using (12) and $y = Y_t/(N_t A_t^d)$, I obtain the vertical R&D condition

$$\frac{\delta \lambda_v \Gamma \alpha (1-\alpha) v^{\delta - 1} y}{\rho - g_L + \frac{g_A}{\sigma} + \frac{g\alpha}{1-\alpha}} = 1 - s_v, \tag{24}$$

which specifies the values of h, v and y that are consistent with vertical R&D profit-maximization in a balanced growth equilibrium (given (20), (21) and (22)). I assume that $\rho > g_L$ to guarantee that the denominator in (24) is always positive.

3.5 The Horizontal R&D Condition

Having just derived Π_{vt}, it follows from (8), (10), (12) and (18) that the corresponding horizontal R&D condition is

$$\frac{\gamma\lambda_h\alpha(1-\alpha)h^{\gamma-1}y}{\rho - g_L + \frac{g_A}{\sigma} + \frac{g\alpha}{1-\alpha}} = 1 - s_h, \tag{25}$$

which specifies the values of h, v and y that are consistent with horizontal R&D profit-maximization in a balanced growth equilibrium (given (20), (21) and (22)).

3.6 General R&D Subsidies

The population growth condition (23), the vertical R&D condition (24) and the horizontal R&D condition (25) represent a system of 3 non-linear equations in 3 unknowns (h, v and y) that must be simultaneously satisfied by any balanced growth equilibrium (given (20), (21) and (22)). Solving this system yields the main result in the paper:

Theorem 1 *The model has a unique balanced growth equilibrium. Focusing on this equilibrium, a permanent increase in the general R&D subsidy rate* $s = s_h = s_v$
(i) decreases the long-run economic growth rate g *if* $d > 1/(1-\alpha)$ *and* $\delta > \gamma$*,*
(ii) increases the long-run economic growth rate g *if* $d > 1/(1-\alpha)$ *and* $\gamma > \delta$*,*
(iii) decreases the long-run economic growth rate g *if* $1/(1-\alpha) > d$ *and* $\gamma > \delta$*,*
(iv) increases the long-run economic growth rate g *if* $1/(1-\alpha) > d$ *and* $\delta > \gamma$*, and*
(v) has no effect on g *if either* $d = 1/(1-\alpha)$ *or* $\delta = \gamma$*.*

Theorem 1 represents a generalization of the results derived in Howitt (1999). Consistent with Theorem 1, Howitt showed that R&D subsidies promote long-run economic growth when $d = 1$ and $\delta = 1 > \gamma$ (case (iv)) and that R&D subsidies have no long-run growth effects when $\delta = \gamma < 1$ (case (v)). Theorem 1 goes further in completely characterizing when R&D subsidies promote long-run economic growth (cases (ii) and (iv)) and in showing that for a wide range of plausible parameter values, R&D subsidies retard long-run economic growth (cases (i) and (iii)).

The proof of Theorem 1 is lengthy and does not shed real light on why R&D subsidies can either promote or retard long-run economic growth. Thus, the proof is relegated to the Appendix and in the next subsection, I focus on developing an intuitive explanation for the results reported in Theorem 1.

3.7 An Intuitive Explanation

I begin by examining two polar extreme cases. First, I explore the model's properties when firms only engage in vertical R&D activities ($H_t = 0$) and then when firms only engage in horizontal R&D activities ($V_t = 0$).

The equation $H_t = 0$ holds if it is not profitable for firms to engage in any horizontal R&D activities ($\gamma = 1$ and λ_h is sufficiently small). Then (22) and (23) together imply that the long-run rate of economic growth with only vertical R&D is given by

$$g_A = \frac{g_L}{d-1}. \tag{26}$$

Note that the R&D subsidy rate s does not appear in (26). Thus, a permanent increase in the R&D subsidy rate s has no effect on the long-run rate of economic growth, which is completely determined by exogenous parameters like the population growth rate g_L. In the special case where $H_t = 0$, the model has the same qualitative properties as Segerstrom's (1998) model of growth driven by vertical innovation.

The equation $V_t = 0$ holds if it is not profitable for firms to engage in any vertical R&D activities ($\delta = 1$ and λ_v is sufficiently small). Then (22) and (23) together imply that the long-run rate of economic growth with only horizontal R&D is given by

$$(1-\alpha)g_N = \frac{(1-\alpha)g_L}{\alpha}. \tag{27}$$

Note that the R&D subsidy rate s does not appear in (27) either. Thus, a permanent increase in the R&D subsidy rate s has no effect on the long-run rate of economic growth, which is completely determined by exogenous parameters like the population growth rate g_L. In the special case where $V_t = 0$, the model has the same qualitative properties as Jones's (1995) model of growth driven by horizontal innovation.

Comparing (26) and (27), it immediately follows that $(1-\alpha)g_N > g_A$ if and only if $d > 1/(1-\alpha)$. Thus the d-parameter inequalities in Theorem 1 have simple economic interpretations. The condition $d > 1/(1-\alpha)$ means that economic growth is faster when firms only engage in horizontal R&D activities and the condition $1/(1-\alpha) > d$ means that economic growth is faster when firms only engage in vertical R&D activities.

The next step to understanding Theorem 1 is to develop an economic interpretation of the $\gamma\delta$-parameter inequalities. Suppose that we start off in a balanced growth equilibrium where $h = v$ and $\delta > \gamma$ holds. The marginal cost and marginal benefit curves associated with both horizontal and vertical R&D are illustrated in Figure 1. The initial marginal cost of both horizontal and vertical R&D is given by $1 - s$ and is illustrated in Figure 1 by the horizontal line. An increase in the general R&D subsidy rate s causes the marginal cost line to shift down (as is illustrated by the dashed line). Referring back to (15) and (18), both marginal benefit curves are downward sloping and given $\delta > \gamma$, the marginal benefit of horizontal R&D curve is steeper than the marginal benefit of vertical R&D curve. Thus, an increase in the R&D subsidy rate s leads to a smaller increase in h (the movement from point A to point B) than in v (the movement from point A to point C). The parameter condition $\delta > \gamma$ means that there are greater diminishing returns to horizontal R&D effort than to vertical R&D effort and under these circumstances, R&D subsidies promote

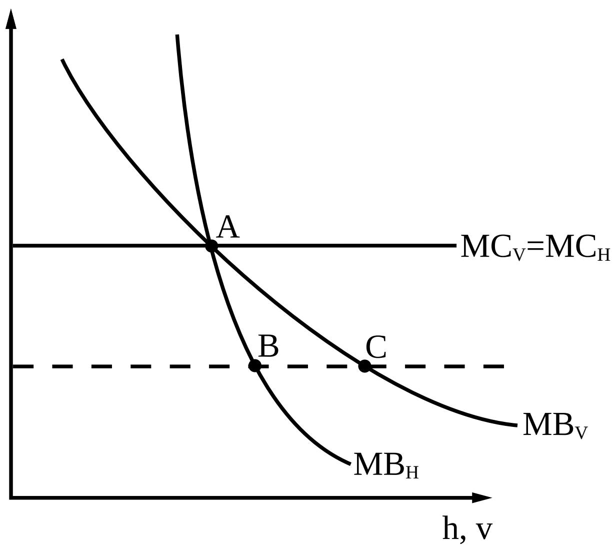

Figure 1: The effect of a general R&D subsidy when $\delta > \gamma$

vertical R&D effort to a greater extent than horizontal R&D effort. The parameter condition $\delta < \gamma$ has the opposite economic interpretation: this condition means that there are greater diminishing returns to vertical R&D effort than to horizontal R&D effort and under these circumstances, R&D subsidies promote horizontal R&D effort to a greater extent than vertical R&D effort.

It is now time to put the pieces of the puzzle together and develop an intuitive understanding of the results in Theorem 1. The effects of a permanent increase in the general R&D subsidy rate s are illustrated in Figure 2. In (g_A, g_N) space, it follows from (22) that each iso-growth curve is a downward sloping line with slope $\frac{-1}{1-\alpha}$ and it follows from (23) that the population growth condition is a downward sloping line with slope $\frac{-(d-1)}{\alpha}$. Thus, the slope of the population growth growth condition exceeds the slope of each iso-growth line (in absolute value) if and only if $d > \frac{1}{1-\alpha}$. Figure 2 illustrates the $d > \frac{1}{1-\alpha}$ case.

Starting from a balanced growth equilibrium path, the initial effect of a general R&D subsidy increase is to encourage firms to devote more resources to both horizontal and vertical R&D. Greater R&D effort in turn leads to faster rates of both horizontal and vertical innovation. When $\gamma < \delta$, that is, there are greater diminishing returns to horizontal R&D, the R&D subsidy increase encourages vertical R&D effort to a greater extent than horizontal R&D effort and the quality growth rate g_A jumps up more than the variety growth rate g_N. On the other hand, when $\delta > \gamma$ and there are greater diminishing returns to verti-

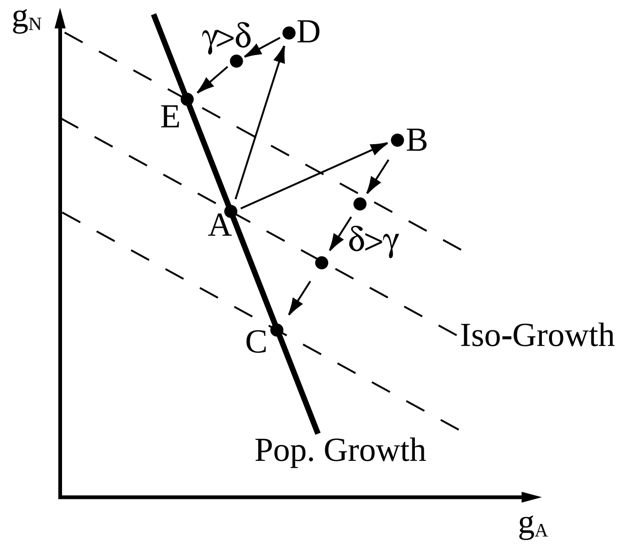

Figure 2: Two adjustment processes leading to new balanced growth equilibria

cal R&D, the R&D subsidy increase encourages horizontal R&D effort to a greater extent than vertical R&D effort. Then the variety growth rate g_N jumps up more than the quality growth rate g_A. In either case though, the increase in R&D effort and the corresponding increase in the rate of technological change means that the complexity of the problems researchers are trying to solve grows over time at a faster than usual rate. Thus, one should expect the overall productivity of researchers in discovering new products to gradually fall over time. The process of gradually declining innovation rates continues until the economy reaches an outcome that is sustainable in the long-run, that is, until the population growth condition has been reached.

When $\gamma < \delta$, the entire adjustment process in response to an increase in the R&D subsidy rate s is illustrated in Figure 2 by the initial jump from equilibrium point A to point B and then by the gradual fall in innovation rates from point B to the final equilibrium point C. Because horizontal R&D is subject to greater diminishing returns, the R&D subsidy increase initially leads to a larger increase in vertical R&D effort and the quality growth rate g_A jumps up more than the variety growth rate g_N. Then, with the complexity of both horizontal and vertical R&D problems increasing over time at a faster than usual rate, the horizontal and vertical innovation rates gradually fall to point C on the population growth condition. The long-run effect of a permanent R&D subsidy increase is to promote vertical innovation g_A at the expense of horizontal innovation g_N. In the illustrated case $d >$

$1/(1-\alpha)$, the vertical R&D-only equilibrium is associated with a lower economic growth rate than the horizontal R&D-only equilibrium. Thus, by promoting vertical innovation g_A at the expense of horizontal innovation g_N, the R&D subsidy increase has the long-run effect of moving the economy closer to the vertical R&D-only equilibrium and thus serves to retard economic growth. As illustrated, the final equilibrium point C is on a lower iso-growth line than the initial equilibrium point A.

When $\delta < \gamma$, the above-mentioned story about how the economy adjusts over time gets reversed. In this case, the adjustment process in response to an increase in the R&D subsidy rate β is illustrated by the initial jump from equilibrium point A to point D and then by the gradual fall in innovation rates from point D to the final equilibrium point E. Because vertical R&D is subject to greater diminishing returns, the R&D subsidy increase initially leads to a larger increase in horizontal R&D effort and the variety growth rate g_N jumps up more than the quality growth rate g_A. Then, with the complexity of both horizontal and vertical R&D problems increasing over time at a faster than usual rate, the horizontal and vertical innovation rates gradually fall to point E on the population growth condition. The long-run effect of a permanent R&D subsidy increase is to promote horizontal innovation g_N at the expense of vertical innovation g_A . In the illustrated case $d > 1/(1-\alpha)$, the vertical R&D-only equilibrium is associated with a lower economic growth rate than the horizontal R&D-only equilibrium. Thus, by promoting horizontal innovation g_N at the expense of vertical innovation g_A, the R&D subsidy increase has the long-run effect of moving the economy closer to the horizontal R&D-only equilibrium and thus serves to promote economic growth. As illustrated, the final equilibrium point E is on a higher iso-growth line than the initial equilibrium point A.

The above-mentioned intuition only needs to be slightly modified to deal with the $1/(1-\alpha) > d$ case. Then the vertical R&D-only equilibrium is associated with a higher economic growth rate than the horizontal R&D-only equilibrium. When the R&D subsidy increase has the effect of promoting vertical innovation g_A at the expense of horizontal innovation g_N ($\delta > \gamma$), the R&D subsidy increase has the long-run effect of moving the economy closer to the vertical R&D-only equilibrium and thus serves to promote economic growth. On the other hand, when the R&D subsidy increase has the effect of promoting horizontal innovation g_N at the expense of vertical innovation g_A ($\delta < \gamma$), the R&D subsidy increase has the long-run effect of moving the economy closer to the horizontal R&D-only equilibrium and thus serves to retard economic growth.

It is now clear why general R&D subsidies promote long-run economic growth in Howitt (1999). Howitt assumes that vertical R&D is subject to constant returns whereas horizontal R&D is subject to diminishing returns ($\gamma < \delta = 1$), which implies that general R&D subsidies have the long-run effect of promoting vertical innovation at the expense of horizontal innovation. Furthermore, Howitt assumes that $d = 1 < 1/(1-\alpha)$, which implies that the horizontal R&D-only equilibrium is associated with a finite economic growth rate and the vertical R&D-only "equilibrium" is associated with a infinitely high economic growth rate (substitute $d = 1$ into (26)). Under these circumstances, general R&D subsidies promote long-run economic growth by encouraging vertical innovation at the expense of horizontal innovation and moving the economy closer to the vertical R&D-only "equilibrium" with the infinitely high economic growth rate.

3.8 Targeted R&D Subsidies

Given the above-mentioned intuition, it would appear that the government can promote economic growth even when $\gamma = \delta$ by using appropriately chosen targeted R&D subsidies to guarantee that the right type of innovation increases in the long-run. For example, when $d < 1/(1-\alpha)$ and the vertical R&D-only equilibrium is associated with a higher growth rate than the horizontal R&D-only equilibrium, the government could promote economic growth by only subsidizing vertical R&D activities. This is indeed the case, as the following theorem establishes:

Theorem 2 *A permanent increase in the horizontal R&D subsidy rate* s_h
(i) decreases the long-run economic growth rate g *if* $d < 1/(1-\alpha)$,
(ii) increases the long-run economic growth rate g *if* $d > 1/(1-\alpha)$, *and*
(iii) has no effect on g *if* $d = 1/(1-\alpha)$.
A permanent increase in the vertical R&D subsidy rate s_v
(iv) increases the long-run economic growth rate g *if* $d < 1/(1-\alpha)$,
(v) decreases the long-run economic growth rate g *if* $d > 1/(1-\alpha)$, *and*
(vi) has no effect on g *if* $d = 1/(1-\alpha)$.

Proof: See the Appendix.

Theorem 1 states that if either $d = 1/(1-\alpha)$ or $\delta = \gamma$, then general R&D subsidies do not have long-run growth effects. Given Theorem 2, the more fundamental of these two parameter conditions is $d = 1/(1-\alpha)$. If $d = 1/(1-\alpha)$, then no R&D subsidies (general or targeted) have long-run growth effects, whereas if $\delta = \gamma$ and $d \neq 1/(1-\alpha)$, then targeted R&D subsidies have long-run growth effects.

Theorem 2 sheds important light on otherwise puzzling results in the R&D-driven growth literature. For example, both Dinopoulos and Thompson (1998, pp. 322) and Peretto (1998, pp. 300-302) find that vertical R&D subsidies promote growth and horizontal R&D subsidies retard growth.[7] In their models, the horizontal R&D-only equilibrium is associated with a finite growth rate and the vertical R&D-only "equilibrium" is associated with an infinite growth rate (the growth rate gradually increases over time without upper bound due to population growth and the scale effect property). Their assumptions about R&D correspond to the special case $d = 1$ in this paper or the $d < 1/(1-\alpha)$ case in Theorem 2. Thus, they implicitly assume that vertical R&D is the higher growth activity. As a consequence, subsidies targeted at the higher growth activity (vertical R&D) promote growth and subsidies targeted at the lower growth activity (horizontal R&D) retard growth.[8]

4 Related Literature

The research that is most closely related to this paper is by Li (1999), who also studies the general properties of two sector R&D-driven growth models. To properly appreciate Li's different findings, it is helpful to first examine his different assumptions about R&D. Since Li assumes constant returns to both horizontal and vertical R&D, in making comparisons between the two models, I will restrict attention to this paper's special case $\delta = \gamma$. Like in this paper, Li assumes that there are positive knowledge spillovers between horizontal and vertical R&D activities. Both horizontal and vertical innovations increase the productivity

[7]Both Dinopoulos and Thompson (1998) and Peretto (1998) use different terminology than in this paper. Dinopoulos and Thompson refer to "horizontal R&D" as "new product line development" and they refer to "vertical R&D" as "quality-enhancing R&D." Peretto refers to a"horizontal R&D subsidy" as a"R&D subsidy granted to entrants" and refers to a "vertical R&D subsidy" as a"R&D subsidy granted to incumbents."

[8]Howitt (1999) does not explicitly consider the growth effects of targeted R&D subsidies but it is straightforward to verify that his model has the same targeted subsidy properties as in Dinopoulos and Thompson (1998) and Peretto (1998), and for the same reasons.

of all R&D workers. In this paper, these positive knowledge spillovers are captured by the terms $\lambda_v(Y_t/N_t)^{1-\delta}$ and $\lambda_h(Y_t/N_t)^{1-\delta}$ in (16) and (19), respectively, since innovations increase Y_t/N_t over time. In contrast, Li assumes that the corresponding positive knowledge spillover terms are

$$\lambda_v N_t^{\phi_v} A_t^{\delta_v} \qquad \lambda_h N_t^{\phi_h} A_t^{\delta_h}$$

for vertical and horizontal R&D, respectively, where N_t represents knowledge created through horizontal innovation, A_t represents knowledge created through vertical innovation, and parameters $\phi_v, \delta_v, \phi_h, \delta_h > 0$ are the weights attached to each type of knowledge spillover.

While one would expect that knowledge spillovers are stronger for some R&D activities than for others ($\lambda_v \neq \lambda_h$), allowing for differences in the exponents of the knowledge spillover functions ($\phi_v \neq \phi_h$ or $\delta_v \neq \delta_h$) has strong long-run implications. To see this clearly, consider what happens in the long-run when $\delta_v = \delta_h$ and $\phi_v > \phi_h$. Since both N_t and A_t grow without bound as time passes,

$$\lim_{t\to\infty} \frac{\lambda_v N_t^{\phi_v} A_t^{\delta_v}}{\lambda_h N_t^{\phi_h} A_t^{\delta_h}} = \lim_{t\to\infty} \frac{\lambda_v}{\lambda_h} N_t^{\phi_v - \phi_h} = \infty.$$

Thus, allowing for even the slightest increase in ϕ_v above ϕ_h implies that in the long-run, the knowledge spillovers from horizontal innovations to vertical R&D are *infinitely* larger than the corresponding knowledge spillovers to horizontal R&D.

By studying the properties of his model when $\phi_v \neq \phi_h$ or $\delta_v \neq \delta_h$, Li implicitly focuses on the case of *explosive* knowledge spillover differences between horizontal and vertical R&D. Assuming explosive knowledge spillover differences between horizontal and vertical R&D, Li shows that, except for a knife-edge set of parameter values, neither general nor targeted R&D subsidies have any long-run growth effects. Growth is "semi-endogenous" just like in the one-R&D sector models of Jones (1995) and Segerstrom (1998).

Since

$$\lim_{t\to\infty} \frac{\lambda_v (Y_t/N_t)^{1-\delta}}{\lambda_h (Y_t/N_t)^{1-\delta}} = \frac{\lambda_v}{\lambda_h} < \infty,$$

this paper focuses on the case of *nonexplosive* knowledge spillover differences between horizontal and vertical R&D. Assuming nonexplosive knowledge spillover differences, I find that, except for a knife-edge set of parameter values, both general and targeted R&D

subsidies have long-run growth effects. Growth is “endogenous” as in Howitt (1999) but R&D subsidies often retard (instead of promote) long-run economic growth.

5 Conclusions

This paper presents a model where there are two engines of growth, a vertical dimension and a horizontal dimension. Firms engage in vertical R&D to improve the quality of existing products and firms engage in horizontal R&D to increase the number of industries in the economy (create entirely new products). Firms that innovate and become industry leaders earn temporary monopoly profits as a reward for their R&D efforts. Thus, the “process of creative destruction” originally described by Schumpeter (1942) characterizes industry dynamics.

The model has a unique balanced growth equilibrium in which the fraction of the labor force that engages in R&D is constant over time. Due to positive population growth, the expected discounted profits generated by both horizontal and vertical innovations grow over time. However counterbalancing these trends are the forces of increasing complexity; as technology advances, the resource costs of further advances also increase. In the balanced growth equilibrium, the rising rewards for innovating induces firms to hire more R&D workers but these effects are exactly balanced by the falling productivity of researchers, resulting in constant rates of both horizontal and vertical innovation. Thus, the model's properties are roughly consistent with the evidence presented in Jones (1995) to refute R&D-driven endogenous growth theory. Also the model's prediction of a constant share of GDP allocated to R&D is roughly consistent with the U.S. postwar data presented in Howitt (1999) and the model can account for the declining patents-per-researcher ratio evidence reported in Kortum (1997).

The focus of the paper is on understanding the long-run effects of R&D subsidies. Starting from a balanced growth equilibrium, when there is a permanent increase in the R&D subsidy rate, firms immediately respond by increasing their horizontal and vertical R&D expenditures. However, with firms devoting more resources to R&D, technological complexity also increases more rapidly. Researchers exhaust the supply of simpler problems more quickly and find themselves wrestling with more complicated research problems. Both horizontal and vertical innovation rates gradually fall over time in response to the steady decline in the productivity of R&D workers. Since the rates of horizontal and vertical innovation are ultimately constrained by the population growth rate and the R&D subsidy increase does not change the population growth rate, horizontal and vertical innovation rates continue to fall over time as long as they are both higher than in the initial (pre-R&D subsidy increase) balanced growth equilibrium. Thus, in the long-run, any change in the horizontal innovation rate is matched by a corresponding opposite change in the vertical innovation rate. R&D subsidies never permanently increase both the horizontal and vertical innovation rates in the economy. This is the fundamental new insight in the paper and the key to understanding the long-run growth effects of R&D subsidies.

Given that R&D subsidies have long-run growth effects by promoting one engine of growth (horizontal or vertical innovation) at the expense of the other, these long-run growth effects hinge on which engine is stronger and which engine is promoted by R&D subsidies in the long-run. In general, one engine will be stronger than the other. If R&D subsidies promote the stronger engine, then R&D subsidies increase the long-run rate of economic growth. However, if R&D subsidies promote the weaker engine, then R&D subsidies decrease the long-run rate of economic growth.

If R&D difficulty increases rapidly (slowly) as product quality improves in the typical industry, then the economic growth rate that can be sustained in the long-run when firms only do vertical R&D is low (high), as is formally shown in Segerstrom (1998). On the other hand, the rate at which R&D difficulty increases as product quality improves is irrelevant to determining the economy's long-run economic growth rate when firms only do horizontal R&D. Thus, if R&D difficulty increases rapidly, then horizontal innovation is the stronger engine of growth and if R&D difficulty increases slowly, then vertical innovation is the stronger engine of growth.

The engine that is promoted by general R&D subsidies in the long-run depends on how the static returns to horizontal and vertical R&D expenditure differ. In addition and seperate from the strength of the engine, each dimension (horizontal or vertical) has its own degree of static diminishing returns to R&D expenditure. In the long-run, a general

R&D subsidy increases innovation in the dimension in which diminishing returns set in more slowly. If this dimension happens to correspond to the dimension with the stronger growth engine, then a general R&D subsidy increases long-run growth: this is the setup that Howitt (1999) assumed. However, there is no a priori reason for this to occur. If the dimension with weaker diminishing returns also possesses the weaker engine of growth, then a general R&D subsidy decreases long-run growth.[9]

To summarize, general R&D subsidies decrease long-run growth if they promote vertical innovation (horizontal R&D expenditure is subject to greater diminishing returns) and horizontal innovation is the stronger engine of growth (R&D difficulty increases rapidly as product quality improves). General R&D subsidies also decrease long-run growth if they promote horizontal innovation (vertical R&D expenditure is subject to greater diminishing returns) and vertical innovation is the stronger engine of growth (R&D difficulty increases slowly as product quality improves). General R&D subsidies increase long-run growth in the opposite cases. For example, because Howitt (1999) focused on the case where general R&D subsidies promote vertical innovation (horizontal R&D expenditure is subject to greater diminishing returns) and vertical innovation is the stronger engine of growth (R&D difficulty increases slowly as product quality improves), he concluded that general R&D subsidies increase long-run growth.

Although the focus of this paper is on the long-run growth effects of R&D subsidies, it is worth pointing out that based on the transitional dynamics illustrated in Figure 2, R&D subsidies always appear to promote economic growth in the short run. Thus, even in the cases where R&D subsidies retard long-run economic growth, if convergence to the balanced growth path is very slow, it may take many decades before the economy begins to experience the negative growth effects of permanently higher R&D subsidies. How fast convergence occurs in this model is an open question that needs to be explored in future research.

[9] With targeted R&D subsidies, the situation is simpler. Independent of how the static returns to horizontal and vertical R&D expenditure differ, R&D subsidies that are targeted at the dimension with the stronger growth engine increase long-run growth and R&D subsidies that are targeted at the dimension with the weaker growth engine decrease long-run growth. This is the key to understanding the otherwise puzzling subsidy results in Dinopoulos and Thompson (1998) and Peretto (1998).

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Appendix

The Distribution of Relative Productivities: The following proof is essentially the same as in the appendix to Howitt (1999) but is spelled out in more detail. Let $G(\cdot, t)$ denote the cumulative distribution of (absolute) productivity parameters A_{it} at time t. Pick any $A > 0$ that was the leading-edge productivity parameter at some time $t_0 \geq 0$ and define $\Phi(t) \equiv G(A, t)$. Then $\Phi(t_0) = 1$ since no industry can have a productivity parameter larger than that of the leading-edge productivity parameter at time t_0, which by construction is A. It also follows that

$$\dot{\Phi}(t) + \phi_t \Phi(t) = 0 \tag{28}$$

holds for all $t \geq t_0$. To understand this differential equation, first note that since horizontal innovations represent random draws from the distribution of productivity parameters, they do not change the distribution of productivity parameters and thus can be ignored when characterizing the time path of Φ. Next note that after time t_0, the rate at which vertical innovations cause the mass of industries behind A to fall is the overall flow of vertical innovations occurring in industries currently behind A. There are $\Phi(t)$ such industries and the Poisson arrival rate of vertical innovations in each one of these industries is ϕ_t.

Taking into acount the initial value condition $\Phi(t_0) = 1$, the unique solution to the first order linear differential equation (28) is

$$\Phi(t) = e^{-\int_{t_0}^{t} \phi_s ds} \tag{29}$$

for all $t \geq t_0$. Equation (4) represents another first order linear differential equation which, taking into account the initial condition $A_{t_0} = A$, has a unique solution

$$A_t = A e^{\sigma \int_{t_0}^{t} \phi_s ds}. \tag{30}$$

for all $t \geq t_0$.

Let $a \equiv A/A_t$. Then (29) and (30) together imply that $\Phi(t) \equiv \Pr(A_{it} \leq A) = (A/A_t)^{1/\sigma}$ for all $A \geq A_0$, which can be alternatively expressed as

$$\text{Prob}(a_{it} \leq a) = F(a) \equiv a^{1/\sigma} \tag{31}$$

for all $a \geq A_0/A_t$. As t converges to $+\infty$, A_0/A_t converges to zero. Thus, the distribution of relative productivities converges monotonically over time to the invariant distribution $F(\cdot)$.

Lemma 1 *The model has a unique balanced growth equilibrium. Focusing on this equilibrium, a permanent increase in the general R&D subsidy rate s*
(i) permanently increases the fraction of GDP allocated to vertical R&D v but decreases the long-run product quality growth rate g_A if $\gamma > \delta$,
(ii) permanently increases the fraction of GDP allocated to vertical R&D v but does not change the long-run product quality growth rate g_A if $\gamma = \delta$, and
(iii) permanently increases the fraction of GDP allocated to horizontal R&D h but decreases the long-run product variety growth rate g_N if $\gamma < \delta$.

Proof of Lemma 1: The vertical R&D condition (24) and the horizontal R&D condition (25) can only simultaneously hold if

$$\frac{\delta \lambda_v v^{\delta-1}\Gamma}{1-s_v} = \frac{\gamma \lambda_h h^{\gamma-1}}{1-s_h}. \tag{32}$$

This “mutual R&D” condition has a natural economic interpretation. Changes in the economic environment that increase the relative reward for innovating contribute to increasing both the fraction of GDP devoted to horizontal R&D h and the fraction of GDP devoted to vertical R&D v, that is, h and v tend to rise or fall together.

Using (20) and (21) , (32) can be written more compactly as

$$g_N = c_1 g_A v^{\epsilon}, \tag{33}$$

where $c_1 \equiv \frac{\lambda_h}{\sigma\lambda_v}\left(\frac{\lambda_h\gamma(1-s_v)}{\lambda_v\Gamma\delta(1-s_h)}\right)^{\gamma/(1-\gamma)}$ and $\epsilon \equiv \frac{1-\delta}{1-\gamma}\gamma - \delta$. Substituting (20), (21) and (33) into (23) and (24), the population growth condition becomes

$$g_L = g_A\left(d - 1 + \alpha c_1 v^{\epsilon}\right) \tag{34}$$

and the (now general) R&D condition becomes

$$\rho - g_L = g_A \left[\frac{\delta \Gamma \alpha(1-\alpha)}{\sigma(1-s_v)v} - \frac{1}{\sigma} - \frac{\alpha}{1-\alpha} - \alpha c_1 v^{\epsilon} \right]. \tag{35}$$

Equations (34) and (35) represent a system of two equations in two unknowns (v and g_A) that can be solved for a balanced growth equilibrium. These equations are graphed in Figure 3 assuming that $\gamma > \delta$ (which implies that $\epsilon > 0$). Then the population growth

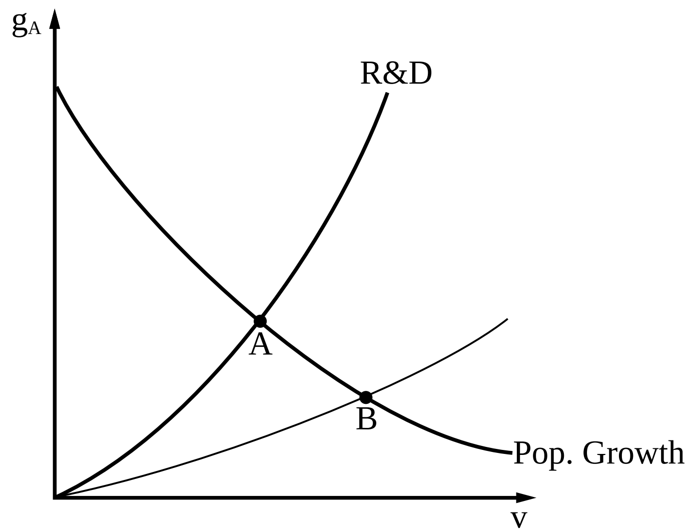

Figure 3: The effect of a general R&D subsidy when $\gamma > \delta$

condition (34) is unambiguously downward sloping and has a strictly positive vertical intercept, whereas the R&D condition (35) is unambigously upward sloping and goes through the origin. As illustrated in Figure 3, there is a unique intersection of these two curves at point A, which pins down the balanced growth equilibrium values of v and g_A. With these values determined, (33) pins down g_N, (20) pins down y, and then (21) pins down h. Thus the model has a unique balanced growth equilibrium when $\gamma > \delta$. In the special case of $\gamma = \delta$ (not illustrated in Figure 3), the population growth condition (34) is a horizontal line with a strictly positive vertical intercept and the above-mentioned argument continues to imply that the model has a unique balanced growth equilibrium.

The effect of permanently increasing the general R&D subsidy rate $s = s_h = s_v$ is illustrated in Figure 3 by the movement from point A to point B. An increase in s unambiguously causes the R&D condition (35) to shift down, while having no effect on the population growth condition (34). Thus a higher general R&D subsidy increases v (the fraction of GDP devoted to vertical R&D) but decreases g_A (the quality growth rate in the typical industry) if $\gamma > \delta$. If $\gamma = \delta$, then a higher general R&D subsidy increases v but has no effect on g_A, as the population growth condition is a horizontal line and does not shift in response to the increase in s.

If $\gamma < \delta$, then the above-mentioned arguments do not go through smoothly, so I solve the model somewhat differently in this case. Using (20) and (21), the mutual R&D condition (32) can be alternatively expressed as

$$g_A = c_2 g_N h^{\mu}, \tag{36}$$

where $c_2 \equiv \frac{\sigma\lambda_v}{\lambda_h}\left(\frac{\lambda_v\delta\Gamma(1-s_h)}{\lambda_h\gamma(1-s_v)}\right)^{\delta/(1-\delta)}$ and $\mu \equiv \frac{1-\gamma}{1-\delta}\delta - \gamma$. Substituting (20), (21) and (36) into (23) and (25), the population growth condition becomes

$$g_L = g_N\left[(d-1)c_2 h^{\mu} + \alpha\right] \tag{37}$$

and the (now general) R&D condition becomes

$$\rho - g_L = g_N\left[\frac{\gamma\alpha(1-\alpha)}{(1-s_h)h} - c_2 h^{\mu}\left(\frac{1}{\sigma} + \frac{\alpha}{1-\alpha}\right) - \alpha\right]. \tag{38}$$

Equations (37) and (38) represent a system of two equations in two unknowns (h and g_N) that can be solved for a balanced growth equilibrium. These equations are graphed in Figure 4. Given $\gamma < \delta$ and $\mu > 0$, the population growth condition (37) is unambiguously downward sloping and has a strictly positive vertical intercept, whereas the R&D condition (38) is unambigously upward sloping and goes through the origin. As illustrated in Figure 4, there is a unique intersection of these two curves at point A, which pins down the balanced growth equilibrium values of h and g_N. With these values determined, (36) pins down g_A, (21) pins down y, and then (20) pins down v. Thus the model has a unique balanced growth equilibrium when $\gamma < \delta$ as well.

The effect of permanently increasing the general R&D subsidy rate $s = s_h = s_v$ is illustrated in Figure 4 by the movement from point A to point B. An increase in s unambiguously causes the R&D condition (38) to shift down, while having no effect on the population growth condition (37). Thus a higher general R&D subsidy increases h (the fraction of GDP devoted to horizontal R&D) but decreases g_N (the growth rate of the measure of industries) if $\gamma < \delta$. **Q. E. D.**

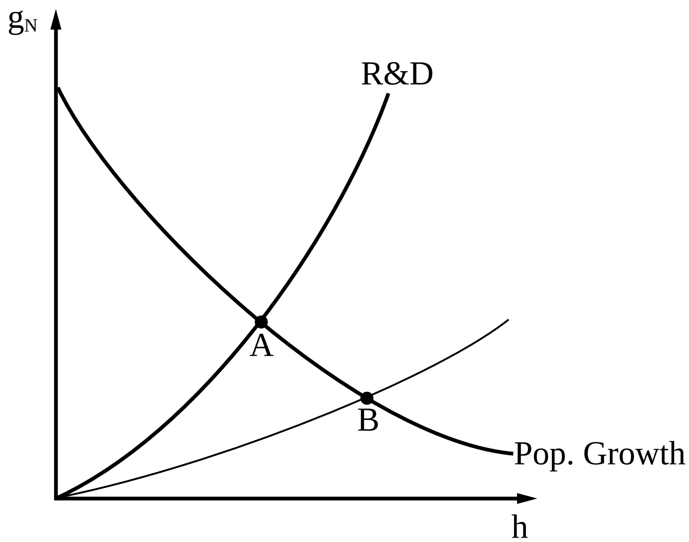

Figure 4: The effect of a general R&D subsidy when $\gamma < \delta$

Proof of Theorem 1: In (g_A, g_N) space, it follows from (22) that each iso-growth curve is a downward sloping line with slope $\frac{-1}{1-\alpha}$ and it follows from (23) that the population growth condition is a downward sloping line with slope $\frac{-(d-1)}{\alpha}$. Thus, the slope of each iso-growth line exceeds the slope of the population growth growth condition (in absolute value) if and only if $\frac{1}{1-\alpha} > d$. The cases $\frac{1}{1-\alpha} > d$ and $d > \frac{1}{1-\alpha}$ are illustrated in Figures 5 and 6, respectively. The mutual R&D condition (given by either (33) or (36)) is also illustrated

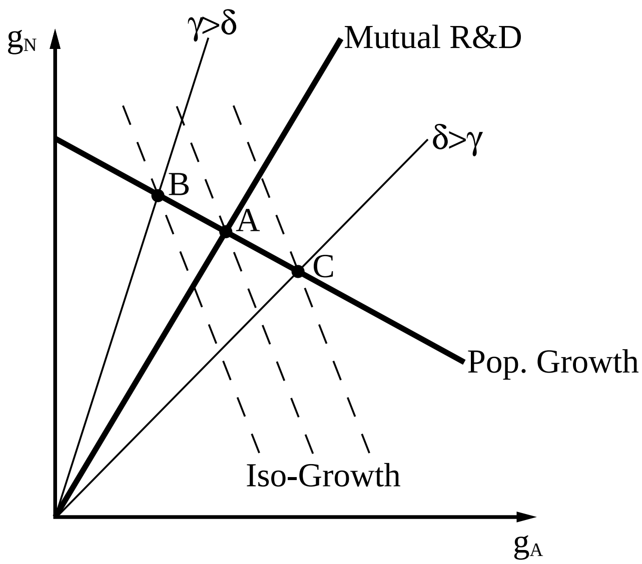

Figure 5: The effect of a general R&D subsidy when $1/(1-\alpha) > d$

and is an upward sloping line that goes through the origin in (g_A, g_N) space, when h and v

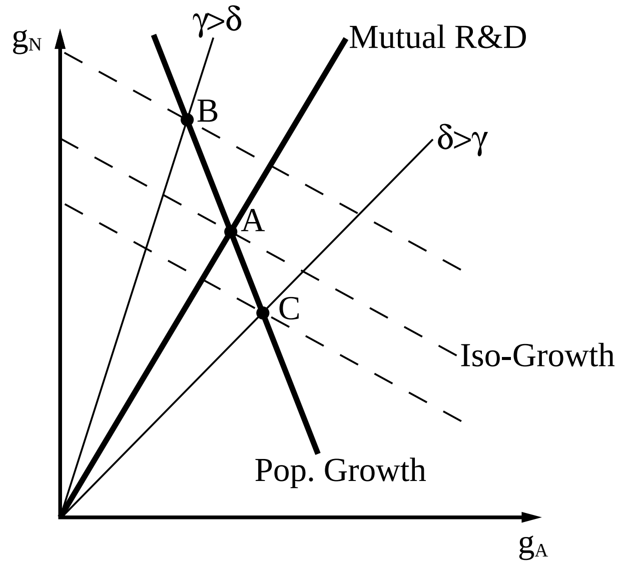

Figure 6: The effect of a general R&D subsidy when $d > 1/(1-\alpha)$

are fixed at their initial equilibrium values. An increase in the R&D subsidy rate s causes the slope of the mutual R&D condition to increase if $\delta < \gamma$ (since the equilibrium value of v increases in (33)) and causes the slope of the mutual R&D condition to decrease if $\delta > \gamma$ (since the equilibrium value of h increases in (36)). In the $1/(1-\alpha) > d$ case illustrated in Figure 5, an increase in the R&D subsidy rate s decreases the long-run growth rate g if $\delta < \gamma$ (the movement from A to B) and increases the long-run growth rate if $\delta > \gamma$ (the movement from A to C). In the $d > 1/(1-\alpha)$ case illustrated in Figure 6, an increase in the R&D subsidy rate s increases the long-run growth rate g if $\delta < \gamma$ (the movement from A to B) and decreases the long-run growth rate if $\delta > \gamma$ (the movement from A to C). R&D subsidies have no growth effects in the $d = 1/(1-\alpha)$ case (not illustrated), since the slope of each iso-growth line coincides with the slope of the population growth condition and R&D subsidies also have no growth effects in the $\delta = \gamma$ case (not illustrated), since the slope of the mutual R&D condition does not change. **Q. E. D.**

Lemma 2 *A permanent increase in the horizontal R&D subsidy rate* s_h
(i) permanently increases the fraction of GDP allocated to horizontal R&D h *and increases the long-run product variety growth rate* g_N *if* $\delta \geq \gamma$*,*
(ii) permanently decreases the fraction of GDP allocated to vertical R&D v *and decreases the long-run product quality growth rate* g_A *if* $\delta \leq \gamma$*.*

A permanent increase in the vertical R&D subsidy rate s_v

(iii) permanently decreases the fraction of GDP allocated to horizontal R&D h and decreases the long-run product variety growth rate g_N if $\delta \geq \gamma$,

(iv) permanently increases the fraction of GDP allocated to vertical R&D v and increases the long-run product quality growth rate g_A if $\delta \leq \gamma$.

Proof of Lemma 2: If $\delta = \gamma$ or $\epsilon = 0$, then (34) is a horizontal line in (v, g_A) space which shifts up when s_v increases and (35) is a upward-sloping curve in (v, g_A) space which shifts down when s_v increases. Thus a permanent increase in s_v (holding s_h fixed) increases both v and g_A. Also (34) shifts down and (35) shifts up when s_h increases, implying that a permanent increase in s_h (holding s_v fixed) decreases both v and g_A.

If $\gamma > \delta$ or $\epsilon > 0$, then (34) is a downward sloping curve in (v, g_A) space which shifts up when s_v increases and (35) is a upward-sloping line in (v, g_A) space which shifts down when s_v increases. Thus a permanent increase in s_v (holding s_h fixed) definitely increases v but more work needs to be done to determine the effect on g_A. Suppose that for some $\gamma > \delta$, an increase in s_v has no effect on g_A. Then (34) implies that $c_1 v^{\epsilon}$ does not change when s_v increases, from which it follows that $[(1 - s_v)v]v^{-\delta/\gamma}$ does not change when s_v increases. But we have already established that v increases in response to an increase in s_v and (35) implies that $[(1 - s_v)v]$ does not change when s_v increases. This yields a contradiction. By the continuity of the model, it follows that g_A must always increase in response to an increase in s_v when $\gamma > \delta$.

Now consider the effects of a permanent increase in s_h when $\gamma > \delta$. Since (34) shifts down and (35) shifts up when s_h increases, v definitely decreases but more work needs to be done to determine the effect on g_A. Suppose that for some $\gamma > \delta$, an increase in s_h has no effect on g_A. Then (34) implies that $c_1 v^{\epsilon}$ does not change when s_h increases, and (35) implies that v does not change, contradicting our earlier finding that v definitely decreases. By the continuity of the model, it follows that g_A must always decrease in response to an increase in s_h when $\gamma > \delta$.

Returning to the case where $\delta = \gamma$ or $\mu = 0$, (37) is a horizontal line in (h, g_N) space which shifts up when β_h increases and (38) is a upward-sloping curve in (h, g_N) space which shifts down when β_h increases. Thus a permanent increase in β_h (holding β_v fixed) increases both h and g_N. Also (37) shifts down and (38) shifts up when β_v increases, implying that a permanent increase in β_v (holding β_h fixed) decreases both h and g_N.

If $\delta > \gamma$ or $\mu > 0$, then (37) is a downward sloping curve in (h, g_N) space which shifts up when s_h increases and (38) is a upward-sloping line in (h, g_N) space which shifts down when s_h increases. Thus a permanent increase in s_h (holding s_v fixed) definitely increases h but more work needs to be done to determine the effect on g_N. Suppose that for some $\delta > \gamma$, an increase in s_h has no effect on g_N. Then (37) implies that $c_2 h^{\mu}$ does not change when s_h increases, from which it follows that $[(1 - s_h)h]h^{-\gamma/\delta}$ does not change when s_h increases. But we have already established that h increases in response to an increase in s_h and (38) implies that $[(1 - s_h)h]$ does not change when s_h increases. This yields a contradiction. By the continuity of the model, it follows that g_N must always increase in response to an increase in s_h when $\delta > \gamma$.

Finally, consider the effects of a permanent increase in s_v when $\delta > \gamma$. Since (37) shifts down and (38) shifts up when s_v increases, h definitely decreases but more work needs to be done to determine the effect on g_N. Suppose that for some $\delta > \gamma$, an increase in s_v has no effect on g_N. Then (37) implies that $c_2 h^{\mu}$ does not change when s_v increases, and (38) implies that h does not change, contradicting our earlier finding that h definitely decreases. By the continuity of the model, it follows that g_N must always decrease in response to an increase in s_v when $\delta > \gamma$. **Q. E. D.**

Proof of Theorem 2: Changes in R&D subsidy rates do not shift the population growth condition (23) but instead induce movements along this downward-sloping line. Comparing (23) and (22), a movement on the population growth condition in the northwest direction (g_N increases and g_A decreases) is growth-promoting if $d > 1/(1 - \alpha)$ and a movement on the population growth condition in the southeast direction (g_N decreases and g_A increases) is growth-promoting if $d < 1/(1 - \alpha)$. Since Lemma 2 completely determines the direction of movement on the population growth condition implied by any targeted R&D subsidy increase, Theorem 2 immediately follows. **Q. E. D.**