The R&D Incentives of Industry Leaders

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Abstract

This paper presents a model to explain why industry leader firms often devote substantial resources to R&D activities and explores the welfare implications of this investment. The key new assumption is that industry leaders can improve their own products more easily than can other firms. When industry leaders have R&D cost advantages, it is optimal for the government to subsidize the R&D expenditures of all firms, subsidize the production expenditures of industry leaders, and tax the profits of new industry leaders. Without government intervention, market forces generate too much creative destruction.

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1 Introduction

One of the striking properties of many R&D-driven endogenous growth models is that industry leaders, the firms with the best products or production processes, do not engage in R&D activities. For example, in Grossman and Helpman [1991], Segerstrom [1991], Aghion and Howitt [1992], Caballero and Jaffe [1993], Stokey [1995], and Kortum [1997], all of the R&D investment that drives economic growth is undertaken by industry followers, firms that are less technologically advanced than industry leaders. In these models, it is assumed that all firms have access to the same R&D technology in each industry. With free entry into R&D races, the expected discounted profits of follower firms from participating in these R&D races equal zero in equilibrium. Since industry leaders cannibalize some of their own business by innovating, they have less to gain from innovating than follower firms. Thus, it is not profit maximizing for industry leaders to devote *any* resources to R&D. Firms invest in R&D to become industry leaders but once they succeed, they rest on their past accomplishments and do not try to improve their own products or production processes.

Although the above-mentioned theoretical argument is logically correct, the implication that industry leaders do not invest in R&D is strongly counterfactual. In Table 1, the 1995 net sales and R&D expenditures of some of the most prominent industry leaders in the U.S. are reported. It is clear from this table that not only do many industry leaders devote resources to R&D but their expenditures can be quite substantial. For example, Intel Corporation has maintained its leadership position in the microprocessor industry for over 2 decades by aggressively investing in research.[1] In 1995, Intel invested 1.3 billion dollars in R&D activities, 8 percent of Intel's net sales. For most of the industry leaders in Table 1, R&D expenditures exceed 6 percent of net sales.

In this paper, we present a model to explain why industry leaders often devote substantial resources to R&D activities and explore the welfare implications of this investment. Like in Grossman and Helpman [1991], we study the properties of a "quality ladders" en-

[1]Since 1971, Intel has introduced seven new microprocessor generations (the 8080, 8086, 286, 386, 486, Pentium and Pentium II chips). See Malone [1995] for a history of the microprocessor industry.

dogenous growth model. In each industry, firms compete in R&D races to discover higher quality products. The growth rate of the economy is determined by the R&D behavior of profit-maximizing firms and government policies can influence this growth rate by changing the incentives of these firms. Our model differs from Grossman and Helpman [1991] in three important respects.

First, to make R&D investment more attractive for industry leaders, we drop a key assumption in Grossman and Helpman [1991]; namely that all firms have access to the same R&D technology in each industry. Instead, we assume that industry leaders have R&D cost advantages over follower firms. Since industry leaders are the only firms using state-of-the-art technologies in their respective industries, it seems natural that it would be easier for them to innovate (improve their own products or production processes) than for other firms.

Second, industry leaders face slightly different environments in the two models. Assumptions about consumer preferences yield unit-elastic industry demand functions in Grossman and Helpman [1991], implying infinitely high pure monopoly prices. In their model, industry leaders practice limit-pricing, that is, they charge the highest prices that they can get away with without encouraging production by other firms. In our model by contrast, assumptions about production yield elastic industry demand functions, implying finite pure monopoly prices. We assume that innovations are sufficiently large quality improvements so that industry leaders practice standard monopoly pricing. In both models, only industry leaders produce in equilibrium but this difference in the elasticity of industry demand turns out to have important welfare implications.

Third, Grossman and Helpman [1991] assume that there are constant returns to R&D effort for individual firms in each industry. We relax this assumption by allowing for diminishing returns to R&D effort at the firm level. One important drawback of assuming constant returns is that, except for a knife-edge set of parameter values, either industry leaders or industry followers do all the research in the economy. Empirical evidence indicates that *both* large and small firms (industry leaders and followers) invest in R&D. For example, according to Scherer [1984, ch. 11], companies with fewer than 1,000 employees were responsible for 47.3 percent of important innovations and companies with over 10,000 employees were responsible for 34.5 percent of important innovations. We show that both industry leaders and followers always participate in R&D races when there are diminishing returns to R&D. Thus, we are mainly interested in the diminishing returns case. Kortum [1993] and Thompson [1996] both report evidence of significant diminishing returns to R&D expenditure at the firm level.

The implications of leader R&D cost advantages have been little explored in the endogenous growth literature. The only other work on this issue is by Barro and Sala-i-Martin [1995, ch. 7]. They argue that as long as industry leaders have the *slightest* R&D cost advantage over follower firms, no follower firm will do research aimed at improving the quality of existing goods. In this paper, we show that this result of Barro and Sala-i-Martin [1995] is not robust; in particular, it depends on their implicit assumption that existing producers (industry leaders in product quality) are able to act as Stackelberg leaders in choosing their R&D expenditures. We assume that all firms make their R&D expenditure choices simultaneously and solve each R&D race for Nash equilibrium behavior.[2] We also study a more general model than Barro and Sala-i-Martin [1995] because we allow for diminishing returns to R&D effort. Following Grossman and Helpman [1991], Barro and Sala-i-Martin [1995] only analyze the constant returns to R&D case.

The rest of this paper is organized as follows: in section 2, the model is presented and in section 3, we analyze its balanced growth equilibrium properties. The welfare properties of the model are examined in section 4 along with an analysis of the subsidies and taxes sufficient for optimal resource allocation over time. Finally, the conclusions reached are discussed in section 5.

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[2]To be precise, Barro and Sala-i-Martin [1995] also assume that both leader and follower firms choose their R&D expenditures at the beginning of each R&D race, that is, simultaneously. Thus to justify their focus on Stackelberg equilibrium behavior, one needs to change some assumptions in their model (see the discussion following Theorem 1 for details).

2 The Model

2.1 Overview

This is a model of an economy where there is a single competitively-produced good that consumers buy. The final consumption good is produced using labor and a continuum of different intermediate inputs indexed by ω on the unit interval [0,1]. Each consumer is endowed with one unit of labor which is inelastically supplied. The total endowment of labor in the economy is denoted by L and does not vary over time.

In each intermediate input industry ω, firms can devote resources to R&D to improve the quality of the industry's intermediate input. By improving on the current best quality intermediate input produced in an industry, a successful R&D firm earns monopoly profits from selling its leading-edge intermediate input to final good producers. Lower quality intermediate input producers are priced out of business in equilibrium. Over time, as the quality of intermediate inputs used in final good production rises, workers become more productive and thus R&D fuels per capita consumption growth.[3]

Each firm maximizes its expected discounted profits, taking into account both the size of the monopoly profit flow from R&D success and its likely duration. If other firms do research, this duration is finite, since each industry leader is eventually driven out of business by further innovation. There is uncertainty associated with research at the industry level creating jumpiness in microeconomic outcomes. Because the probabilities of research success across industries are independent and there is a continuum of industries this jumpiness is not transmitted to macroeconomic variables. Consumers have perfect foresight concerning the aggregate rate of technological change over time and choose their expenditure paths accordingly to maximize their discounted utilities. This is a dynamic general equilibrium model, so all markets clear throughout time.

[3] Scherer [1980, p. 409] cites survey evidence that industrial firms devote 59 percent of their R&D effort to the improvement of existing products. Also, according to Denison [1985], 68 percent of the gain in output per worker in the U.S. between 1929 and 1982 can be credited to advances in scientific and technological knowledge, that is, R&D investment broadly construed.

2.2 Product Markets

In each industry, firms are distinguished by the quality of the inputs they produce. Quality is indexed by j where higher values of j denote higher quality products. At time $t = 0$, the state-of-the-art quality product in each industry has a quality index of $j = 0$ and no firm knows how to produce any higher quality product. To learn how to produce higher quality products, firms in each industry engage in R&D races. When the state-of-the-art quality in an industry has index j, the next winner of a R&D race becomes the sole producer of a product with quality index $j + 1$. Thus at time t, the quality index $j(\omega, t)$ of the highest quality product in industry ω also measures the number of successful product upgrades that have occurred in that industry since $t = 0$. The size of each product upgrade is measured by the parameter $\lambda > 1$.

For final good producing firm i, let $x_i(j, \omega, t)$ denote the amount this firm uses of the intermediate input of quality j produced by industry ω at time t. Firm i also uses labor $L_i(t)$ to produce its final good output $Y_i(t)$ at time t. Each firm i has the same production function

$$Y_i(t) = \int_0^1 AL_i(t)^{1-\alpha} \left[\sum_{j=0}^{j(\omega,t)} \lambda^j x_i(j, \omega, t) \right]^\alpha d\omega, \tag{1}$$

where $0 < \alpha < 1$ and $A > 0$ are given production parameters. With $\lambda > 1$ and λ^j increasing in j, (1) implies that higher quality intermediate inputs make each worker more productive. The summation in (1) only runs up through $j(\omega, t)$ since only those intermediate inputs which have been invented by time t can be used in the production of final goods at time t. If only the highest quality intermediate inputs are used in production, which will be the case in equilibrium, then the production function reduces to

$$Y_i(t) = \int_0^1 AL_i(t)^{1-\alpha} \lambda^{j(\omega,t)\alpha} x_i(\omega, t)^\alpha \, d\omega, \tag{2}$$

where $x_i(\omega, t)$ is the quantity of the highest quality intermediate input from industry ω used by firm i at time t.

Suppose that only state-of-the-art quality intermediate inputs are used to produce final goods and let $p(\omega, t)$ denote the price of the leading-edge intermediate input from industry ω at time t (relative to the final good price which we treat as the numeriare). Using (2), the marginal product of intermediate inputs is obtained by differentiating inside the integral with respect to $x_i(\omega, t)$. Profit maximization by firm i implies that the marginal product of each input must equal its price:

$$A\alpha L_i(t)^{1-\alpha}\lambda^{j(\omega,t)\alpha}x_i(\omega,t)^{\alpha-1} = p(\omega,t). \tag{3}$$

Since all firms face the same input prices and choose the same input ratios, we can aggregate across firms to obtain the aggregate demand $X(\omega, t)$ for the highest quality intermediate input from industry ω at time t:

$$X(\omega,t) = L\left[\frac{A\lambda^{j(\omega,t)\alpha}\alpha}{p(\omega,t)}\right]^{\frac{1}{1-\alpha}}. \tag{4}$$

Each intermediate input is nondurable and entails a unit marginal cost of production (measured in terms of final good output Y). The government subsidizes the production of all intermediate inputs by paying a fraction s_p of each firm's production costs and this subsidy policy is financed by lump-sum taxation. Thus leading-edge intermediate input producers choose their prices to solve the profit maximization problem $\max_{p(\omega,t)}\ X(\omega,t)\left[p(\omega,t)-(1-s_p)\right]$ which yields the usual monopoly price markup

$$p(\omega,t) = \frac{1-s_p}{\alpha}. \tag{5}$$

Note that this monopoly price is constant over time and across industries. Furthermore, if $\frac{1}{\alpha} < \lambda$, then lower quality intermediate input producers are not able to compete even when leading-edge producers charge the unconstrained monopoly price. To simplify the analysis, we assume that $\frac{1}{\alpha} < \lambda$ in the rest of this paper. Then (5) holds in equilibrium and only the highest quality intermediate inputs are used in final good production.

Given (4) and (5), a leading-edge firm with a product of quality j earns the profit flow

$$\pi(j) = \pi_0\lambda^{\frac{j\alpha}{(1-\alpha)}} \tag{6}$$

where $\pi_0 \equiv L\left[\frac{1-\alpha}{\alpha}\right]\left[A\alpha^2\right]^{\frac{1}{1-\alpha}}(1-s_p)^{\frac{-\alpha}{1-\alpha}}$. The profit flow earned by an industry leader is constant during a R&D race and jumps up every time quality is upgraded.

The total resources devoted to intermediate input production at time t are given by

$$X(t) \equiv \int_0^1 X(\omega,t)\,d\omega = \left[\frac{A\alpha^2}{1-s_p}\right]^{\frac{1}{1-\alpha}} LQ(t), \tag{7}$$

where

$$Q(t) \equiv \int_0^1 \lambda^{\frac{j(\omega,t)\alpha}{1-\alpha}} \, d\omega \tag{8}$$

is an intermediate input quality index. Substituting (3) and (5) into (2), and aggregating across firms gives total output at time t

$$Y(t) = \left[\frac{A^{\frac{1}{\alpha}} \alpha^2}{1 - s_p} \right]^{\frac{\alpha}{1-\alpha}} LQ(t). \tag{9}$$

Since Q grows over time due to R&D activities (as will be shown below), output Y also grows over time. This reflects the increasing productivity of workers over time.

2.3 The Consumer Sector

All consumers have identical preferences and live forever. Each consumer maximizes a familiar expression for utility:

$$U \equiv \int_0^\infty \left(\frac{c(t)^{1-\theta} - 1}{1 - \theta} \right) e^{-\rho t} \, dt, \tag{10}$$

where $c(t)$ is the consumer's final good consumption at time t, $\rho > 0$ is the subjective discount rate and $\theta > 0$ is the constant elasticity of marginal utility with respect to consumption. Maximizing (10) subject to the consumer's intertemporal budget constraint yields the usual intertemporal consumer optimization condition

$$\frac{\dot{c}(t)}{c(t)} = \frac{r(t) - \rho}{\theta}, \tag{11}$$

where $r(t)$ is the equilibrium interest rate at time t. All consumers have the same financial assets at each point in time so $c(t)$ also represents per capita consumption and aggregate consumption is given by $C(t) \equiv c(t)L$.

2.4 The R&D Sector

In each industry $\omega \in [0, 1]$, there are two types of firms that can do R&D; the current quality *leader* (the firm that is producing the state-of-the-art quality intermediate input) and *followers* (all other firms in the industry). Leaders and followers make their R&D expenditure decisions simultaneously, independently, and are free to adjust their expenditures at any point in time. There is free entry by follower firms into each R&D race and each follower firm possesses the same R&D technology. Because there is perfect competition among follower firms in each industry, the R&D expenditures of individual follower firms will be negligible, which greatly simplifies our equilibrium and welfare calculations.

Let $I_i(\omega, t)$ and $I_l(\omega, t)$ denote the instantaneous probabilities of R&D success by follower firm i and the current leader in industry ω at time t, respectively. These probabilities of R&D success are independently distributed across firms, industries and over time. Thus $I(\omega, t) \equiv I_l(\omega, t) + \sum_i I_i(\omega, t)$ is the instantaneous probability that some firm will innovate in industry ω at time t.[4]

Let $R_i(\omega, t)$ and $R_l(\omega, t)$ denote the flow of resources devoted to R&D by follower firm i and the current leader in industry ω at time t (measured in units of final good output Y). The leader's instantaneous probability of R&D success is

$$I_l(\omega, t) = \frac{R_l(\omega, t)^\beta}{c_l d^{j(\omega,t)}}, \tag{12}$$

and follower firm i's instantaneous probability of R&D success is

$$I_i(\omega, t) = \frac{R_i(\omega, t)^\beta m^{\beta-1}}{c_f d^{j(\omega,t)}}, \tag{13}$$

where $d > 1$, $c_f > c_l > 0$, and $1 \geq \beta > 0$.

In equation (13), m represents the total number of follower firms that can participate in each R&D race. Since we have assumed free entry into R&D races, $m = +\infty$. However, to avoid unnecessary confusion, we will temporarily assume that m is a positive integer, solve for profit maximizing behavior on the part of each individual follower firm and then look at what happens in the limit as m converges to infinity.

With these R&D technologies for leader and follower firms, each firm's instantaneous probability of R&D success continuously increases as it devotes more resources to R&D. Since $d^{j(\omega,t)}$ is increasing in j, the assumption $d > 1$ implies that R&D projects become more challenging with each step up the quality ladder in any industry. Every time innova-

[4]By instantaneous probability, we mean that $I(\omega, t)\, dt$ is the probability that some firm will innovate during the time interval from t to $t + dt$, where dt is an infinitesimal increment of time. Alternatively stated, R&D success bears a Poisson probability distribution with arrival rate $I(\omega, t)$.

tion occurs in an industry, the instantaneous probabilities of R&D success for both leader and follower firms decline, for any given levels of R&D effort.

When $\beta = 1$, there are constant returns to R&D expenditure. Then the assumption $c_f > c_l > 0$ implies that in each industry, the current leader has a R&D cost advantage over follower firms. Because leaders are already on the technology frontier, it is easier for them to advance the frontier than other firms. Barro and Sala-i-Martin [1995] make the same assumptions about the R&D technologies of leader and follower firms.

When $\beta < 1$, there are diminishing returns to R&D expenditure at the firm level. These diminishing returns can be interpreted as arising because, at any point in time, firms have a limited number of potentially useful ideas (firm-specific knowledge) for improving existing technologies. Then, (13) implies that as m increases to infinity, the total amount of firm-specific knowlege possessed by follower firms in each industry does not change, but each individual follower firm's firm-specific knowledge falls to zero.

We focus in this paper on the *balanced growth* properties of the model when R&D behavior is symmetric across industries ($I_i(\omega, t)$ and $I_l(\omega, t)$ do not vary across industries $\omega \in [0, 1]$, firms $i = 1, 2, ..., m$, or over time). With all follower firms in an industry choosing the same R&D intensity and independence of returns across firms, (13) allows for convenient aggregation:

$$I_f = \frac{R_f(\omega, t)^\beta}{c_f d^{j(\omega,t)}}, \tag{14}$$

where $R_f(\omega, t) = m \cdot R_i(\omega, t)$ is the total resources devoted to R&D by follower firms and $I_f = m \cdot I_i$ is the instantaneous probability of R&D success by all follower firms combined.

2.5 The Resource Constraint

Together (12) and (14) imply that the economy-wide resources devoted to R&D at time t are

$$R(t) = \int_0^1 [R_l(\omega, t) + R_f(\omega, t)] d\omega = \left[(I_l c_l)^{\frac{1}{\beta}} + (I_f c_f)^{\frac{1}{\beta}} \right] \int_0^1 d^{\frac{j(\omega,t)}{\beta}} d\omega. \tag{15}$$

Resources in the economy (measured in terms of final good output Y) can be either used in the production of intermediate inputs, used in the R&D sector, or consumed. Therefore, the economy-wide resource constraint at time t is

$$Y(t) = C(t) + X(t) + R(t). \tag{16}$$

This completes the description of the model.

3 The Balanced Growth Equilibrium

In this section, we analyze the balanced growth equilibrium properties of the model. When per capita consumption grows over time at a constant rate, (11) implies that the market interest rate r must be constant over time. Let $V_l(j)$ denote the expected discounted profits earned by a leader which sells a quality j intermediate input. Likewise, let $V_f(j)$ denote the expected discounted profits earned by a follower when the state-of-the-art quality in its industry is j. To maximize expected discounted profits, both leaders and followers must solve stochastic optimal control problems where the state variable $j(\omega, t)$ in each industry ω is a Poisson jump process with intensity $I_l + I_f$ and magnitude $+1$.

For each leader, the relevant Hamilton-Jacobi-Bellman equation is[5]

$$\begin{aligned} rV_l(j) = \max_{R_l(j)\geq 0} \quad & \{\pi(j) + I_l\left[V_l(j+1) - V_l(j)\right] - (1-s_r)R_l(j) \\ & + I_f\left[V_f(j+1) - V_l(j)\right]\} \end{aligned} \tag{17}$$

Each leader earns the profit flow $\pi(j)$ and incurs the R&D costs $(1 - s_r)R_l(j)$ today. The government subsidizes R&D by paying the fraction s_r of the firm's total R&D expenditure. With instantaneous probability I_l, the leader innovates and learns how to produce a $j + 1$ quality intermediate input. However, with instantaneous probability I_f, a follower firm instead innovates and the leader becomes a follower. (17) states that the maximized expected return on a leader firm's stock must equal the return on an equal-sized investment in a riskless bond.

[5] See Malliaris and Brock [1982, p. 123-124] for the application of stochastic dynamic programming techniques to Poisson jump processes, and Thompson and Waldo [1994, p. 453] for an economic illustration of these techniques.

For each follower firm i, the relevant Hamilton-Jacobi-Bellman equation is

$$rV_f(j) = \max_{R_i(j)\geq 0} \quad \{I_i\left[(1-\tau)V_l(j+1) - V_f(j)\right] - (1-s_r)R_i(j) \\ +(I_0 + I_l)\left[V_f(j+1) - V_f(j)\right]\} \tag{18}$$

where $I_0 \equiv I_f - I_i$ is the R&D intensity by all the other follower firms combined. Each follower incurs the R&D costs $(1-s_r)R_i(j)$ today but earns no profit flow. With instantaneous probability I_i, the follower innovates, becomes a leader and learns how to produce a $j+1$ quality intermediate input. In the event of success the follower is subject to a government-imposed turnover tax equal to a fraction τ of its new market value. However, with instantaneous probability $I_0 + I_l$, some other firm innovates (either the current leader or another follower) and the follower continues to be a follower in the next R&D race. (18) states that the maximized expected return on a follower firm's stock must equal the return on an equal-sized investment in a riskless bond.

From (18), the first order condition for profit maximization for follower firm i is $\beta R_i(j)^{\beta-1} \cdot m^{\beta-1}\left\{(1-\tau)V_l(j+1) - V_f(j)\right\} = (1-s_r)c_f d^j$. This condition along with (13) implies that, in equilibrium, the instantaneous probability of success for follower firms is $I_f = \{\beta(1-\tau)V_l(j+1) - \beta V_f(j)\}^{\frac{\beta}{1-\beta}}\left[(1-s_r)^\beta c_f d^j\right]^{\frac{-1}{1-\beta}}$.

We will henceforth restrict attention to the limiting case where $m = +\infty$ and there is free entry of follower firms in each R&D race. Then the individual contribution of any particular follower firm i to the aggregate innovation rate of all followers, I_f, is negligible and

$$V_f(j) = 0 \text{ for all } j, \tag{19}$$

that is, the market value of each follower firm equals zero at each point in time. Given (19), it also follows that I_f is a constant for all j only if

$$V_l(j+1) = \frac{kd^{\frac{j}{\beta}}}{1-\tau} \text{ for all } j, \tag{20}$$

where k is some positive constant (to be determined).

There are two reasons why the innovation rate for followers may vary across R&D races. First, as the quality of intermediate inputs increases over time, innovating becomes more difficult since d^j is an increasing function of j. This causes the probability of success to fall for any given follower R&D effort level and by itself reduces the innovation rate. Second, firms adjust their R&D expenditures in response to changes in the reward for innovating. In particular, when $V_l(j)$ is an increasing function of j, firms respond to increasing rewards by increasing their R&D expenditures. Both considerations exactly offset each other, which allows the economy to experience balanced growth, when (20) holds.

Taking into account (19) and (20), the first order conditions for profit maximization for both leader and follower firms yield

$$R_l(j) = d^{\frac{j}{\beta}} \left\{ \frac{\beta k(1 - d^{-\frac{1}{\beta}})}{c_l(1 - s_r)(1 - \tau)} \right\}^{\frac{1}{1-\beta}} \tag{21}$$

and

$$R_f(j) = d^{\frac{j}{\beta}} \left\{ \frac{\beta k}{c_f(1 - s_r)} \right\}^{\frac{1}{1-\beta}}. \tag{22}$$

With R&D expenditures given by (21) and (22), the innovation rates of leaders and followers are

$$I_l = \left\{ \frac{\beta k(1 - d^{-\frac{1}{\beta}})}{c_l^{\frac{1}{\beta}}(1 - s_r)(1 - \tau)} \right\}^{\frac{\beta}{1-\beta}} \tag{23}$$

and

$$I_f = \left\{ \frac{\beta k}{c_f^{\frac{1}{\beta}}(1 - s_r)} \right\}^{\frac{\beta}{1-\beta}}. \tag{24}$$

The instantaneous probabilities of success for leader and follower firms defined in (23) and (24) remain constant across industries and over time. This balanced growth occurs because the associated R&D expenditures in (21) and (22) increase over time exactly enough to compensate for the increasing difficulty of creating new product generations.

With the values determined in (20)-(24), the Hamilton-Jacobi-Bellman equation (17) can only hold with equality for all j if

$$d^{\frac{1}{\beta}} = \lambda^{\frac{\alpha}{1-\alpha}}. \tag{25}$$

We assume that (25) holds, since otherwise the model does not have a balanced growth equilibrium. If d is too large to satisfy (25), then the growth rate of the economy gradually falls to zero over time and if d is too small to satisfy (25), then the growth rate of the economy gradually increases over time.[6]

Substituting (8) and (25) into (15), the economy-wide resources devoted to R&D can be written more simply as

$$R(t) = R_0 Q(t), \tag{26}$$

where $R_0 \equiv R(0) = (I_l c_l)^{\frac{1}{\beta}} + (I_f c_f)^{\frac{1}{\beta}}$. Thus, in a balanced growth equilibrium, the resources devoted to R&D grow over time at the same rate as the quality index $Q(t)$. Substituting (7), (9), and (26) into the economy-wide resource constraint (16) yields

$$c(t) = \frac{C(t)}{L} = (Y_0 - X_0 - R_0)\frac{Q(t)}{L}, \tag{27}$$

where $Y_0 \equiv Y(0)$ and $X_0 \equiv X(0)$. It follows that in a balanced growth equilibrium, per capita consumption grows at the same rate as the quality index $Q(t)$.

To determine the growth rate of the quality index, we begin by noting that the same R&D intensity $I \equiv I_l + I_f$ applies to all industries in a balanced growth equilibrium. It follows that $(It)^m e^{-It}/m!$ is the measure of products that are improved exactly m times by time t. Thus $Q(t) = \int_0^1 d^{\frac{j(\omega,t)}{\beta}}\, d\omega = \sum_{m=0}^{\infty} \frac{(It)^m e^{-It} d^{\frac{m}{\beta}}}{m!} = e^{-It} \sum_{m=0}^{\infty} \frac{(Id^{\frac{1}{\beta}} t)^m}{m!} = e^{It(d^{\frac{1}{\beta}}-1)}$, and the growth rate of $Q(t)$ is given by

$$\frac{\dot{Q}(t)}{Q(t)} = (I_l + I_f)(d^{\frac{1}{\beta}} - 1). \tag{28}$$

Since this also equals the growth rate of per capita consumption, (11) implies that the equilibrium interest rate is

$$r = \theta(I_l + I_f)(d^{\frac{1}{\beta}} - 1) + \rho. \tag{29}$$

The equilibrium value of k along with (21) and (22) will determine the equilibrium R&D expenditures by both leader and follower firms in each industry. Substituting (29) and then (19)-(25) into the Hamilton-Jacobi-Bellman equation (17) and rearranging produces the implicit equation that determines this equilibrium value of k:

$$f(k) \equiv ak + bk^{\frac{1}{1-\beta}} = \pi_0, \tag{30}$$

[6](25) is a generalization of the condition (7.22) in Barro and Sala-i-Martin [1995].

where $b \equiv \left(\frac{\beta}{1-s_r}\right)^{\frac{\beta}{1-\beta}} \left\{ \left(\frac{1-d^{-\frac{1}{\beta}}}{c_l(1-\tau)}\right)^{\frac{1}{1-\beta}} \left[\theta(1-\tau)^{\frac{\beta}{1-\beta}} - (1-\beta)\right] + \frac{\theta(1-d^{-\frac{1}{\beta}})+d^{-\frac{1}{\beta}}}{c_f^{\frac{1}{1-\beta}}(1-\tau)} \right\}$ and $a \equiv \frac{\rho d^{-\frac{1}{\beta}}}{1-\tau}$. Given the restrictions on the model parameters, a is always positive but b can be either positive or negative. In the absence of government intervention ($s_r = s_p = \tau = 0$), the condition $b < 0$ is satisfied if $\theta < 1 - \beta$ (the intertemporal elasticity of substitution θ^{-1} is sufficiently high) and c_l is sufficiently small (industry leaders have most of the potentially useful ideas about how to improve their own products). On the other hand, the condition $b \geq 0$ is satisfied if $\theta > 1 - \beta$ or c_f is sufficiently small.

When $b \geq 0$, the properties $f(0) = 0$, $f'(k) > 0$ and $f''(k) > 0$ for all $k > 0$ guarantee that the function f steadily increases without bound as k increases. Thus the equation $f(k) = \pi_0$ has a unique strictly positive solution k and the model has a unique balanced growth equilibrium.

When $b < 0$, the properties $f(0) = 0$, $f'(k) > 0$ for small k, $f'(k) < 0$ for large k and $f''(k) < 0$ imply that the function f is n-shaped. Then for large π_0, $f(k) = \pi_0$ has no solution, implying that the model does not have a balanced growth equilibrium. On the other hand, when π_0 is small, then the $f(k)$ function and the constant function π_0 intersect at two points, implying that there are two candidates for balanced growth equilibrium.

The right hand intersection has perverse comparative steady-state properties. For example, consider a permanent decrease in the population of consumers L that innovative firms sell to, which generates permanently lower profit flows for industry leaders. Such a decrease is associated with faster innovation rates for both leader and follower firms; k increases, implying that I_l and I_f both increase. In fact, when $L = 0$, that is, innovative firms earn zero profit flows because there are no consumers to sell to, the right hand intersection involves a strictly positive k, implying positive innovation rates and positive market values for industry leaders [see (20)]. Clearly, the right hand intersection represents firm behavior that is built on unrealistic expectations about the future and is not a genuine balanced growth equilibrium. In the rest of this paper, we restrict attention to the left hand intersection of the f and π_0 functions when analyzing balanced growth equilibrium behavior. And since the focus of this paper is on balanced growth properties, we restrict attention to parameter values for which a balanced growth equilibrium exists.

When $b \geq 0$, or $b < 0$ and we focus on the left hand intersection, the model shares several properties with earlier endogenous growth models. It is straightforward to verify that the balanced growth innovation rate $I \equiv I_l + I_f$ is higher when production is subsidized (s_p increases), R&D is subsidized (s_r increases), or consumers are less myopic (ρ decreases). Also, an increase in L implies that π_0, the equilibrium value of k, I_l and I_f all increase. Thus the model has the "scale effect" property: an increase in population size leads firms to devote more resources to R&D activities and results in faster economic growth.

Looking at long stretches of human history, Kremer [1993] finds empirical support for the scale effect property. Among societies with no possibility for technological contact, those with larger initial populations experienced faster technological change. However, since 1950, the world population has increased substantially (as has the resources devoted to R&D in advanced countries) without leading to any upward trend in economic growth rates (see Jones [1995]). This recent evidence has lead economists to develop models of R&D-driven growth without the scale effect property. For example, Segerstrom [1997] shows that the scale effect can be removed from the Grossman and Helpman [1991] quality ladders growth model by assuming that R&D difficulty increases with cumulative R&D effort during each R&D race. Also, to remove the scale effect, Schulstad [1993] assumes that knowledge diffusion occurs more slowly in a larger economy, Jones [1995] assumes that increases in the stock of knowledge contribute to the development of new intermediate inputs at a diminishing rate, and Howitt [1997] assumes that firms engage in both quality-improving and variety-creating R&D.

Following any of the above-mentioned approaches to removing the scale effect would significantly increase the complexity of the model and make comparisons with the earlier endogenous growth literature (in particular, Barro and Sala-i-Martin [1995]) more difficult. Furthermore, none of the featured results in this paper appear to be driven by the scale effect property and thus, these results should be qualitatively robust to its removal. For these reasons, we do not adopt any of the above-mentioned approaches to removing the scale effect and leave these as possible future refinements of the model.

We are now in a position to solve for the equilibrium R&D behavior of industry leader and follower firms. Equations (23) and (24) imply that the relative innovation rate of industry leaders is given by

$$\frac{I_l}{I_f} = \left\{ \frac{c_f(1 - d^{-\frac{1}{\beta}})^\beta}{c_l(1-\tau)^\beta} \right\}^{\frac{1}{1-\beta}}. \tag{31}$$

To facilitate comparison with earlier endogenous growth models, we focus first on the limiting case where β converges to one and there are constant returns to R&D. Then, in the absence of government intervention, (31) implies that $\lim_{\beta \to 1} \frac{I_l}{I_f} = +\infty$ if $c_f \frac{d-1}{d} > c_l$ and $\lim_{\beta \to 1} \frac{I_l}{I_f} = 0$ if $c_f \frac{d-1}{d} < c_l$. Thus, we obtain

Theorem 1 *When there are constant returns to R&D ($\beta = 1$) and the government does not intervene ($s_r = s_p = \tau = 0$),*
(i) only follower firms invest in R&D if industry leaders have a relatively small R&D cost advantage [$c_f \frac{d-1}{d} < c_l$], and
(ii) only industry leaders invest in R&D if industry leaders have a relatively large R&D cost advantage [$c_f \frac{d-1}{d} > c_l$].

The key to understanding Theorem 1 is to recognize that industry leaders have less to gain from innovating than follower firms. Referring back to (17), (18), and (19), each leader's reward for innovating is given by $V_l(j+1) - V_l(j)$ whereas each follower's reward for innovating is given by the larger $V_l(j+1) - V_f(j) = V_l(j+1)$. Leaders have less to gain from innovating because they are already earning positive economic profits. When industry leaders have a small R&D cost advantage, this is more than offset by the difference in benefits from R&D success, and only follower firms invest in R&D activities. However, as the R&D cost advantage becomes larger, eventually this R&D cost advantage comes to dominate the difference in benefits from R&D success, behavior switches, and only industry leaders invest in R&D activities.

Theorem 1 represents a brand-new research finding. Analyzing essentially the same model, Barro and Sala-i-Martin [1995] conclude that only industry leaders do research when they have the slightest R&D cost advantage over follower firms ($c_f > c_l$). The reason why Barro and Sala-i-Martin [1995] reach different conclusions about the R&D behavior of firms is that they solve the model for a different type of equilibrium. They implicitly solve each R&D race for a Stackelberg equilibrium where industry leaders (in product quality) are assumed to be able to act as Stackelberg leaders in choosing their R&D expenditures and do just enough R&D to discourage any R&D effort by follower firms. Barro and Sala-i-Martin's focus on Stackelberg equilibrium behavior can be approximately justified if one assumes that in each R&D race, the current industry leader chooses its R&D effort level first, with a short time lag follower firms observe the industry leader's R&D choice, follower firms then choose their own R&D effort levels, and the industry leader must maintain its initial R&D effort level for the duration of the R&D race (the leader cannot choose a new R&D effort level after learning about how other firms are behaving). But in the model that Barro and Sala-i-Martin analyze, the R&D expenditure choices of both leader and follower firms are made simultaneously, and with no follower R&D spending, the R&D expenditure choices of leaders are too large to be profit-maximizing (when industry leaders have a small R&D cost advantage). Given the model structure, it seems more appropriate to solve for Nash equilibrium R&D behavior as we do here. Then the R&D expenditure choices of leaders are profit-maximizing given the equilibrium R&D expenditure choices of followers and the R&D expenditure choices of followers are profit-maximizing given the equilibrium R&D expenditure choices of leaders.

Although the assumption of constant returns to R&D is commonplace is the endogenous growth literature, in the context of the model, it has an undesirable implication. Except for a knife-edge set of parameter values ($c_f \frac{d-1}{d} = c_l$), either leader firms do all the R&D ($c_f \frac{d-1}{d} > c_l$), or follower firms do all the R&D ($c_f \frac{d-1}{d} < c_l$). Empirically, we observe that both leader and follower firms invest in R&D activities. To account for the innovative behavior of both large and small firms, we also study the model's properties when there are diminishing returns to R&D at the individual firm level.

When $\beta < 1$, the balanced growth equilibrium value of k is strictly positive. Thus, (23) and (24) imply that both leaders and followers invest in R&D activities ($I_l > 0$ and $I_f > 0$). Furthermore, like in the constant returns to R&D case, either $I_f > I_l$ or $I_l > I_f$ is possible. Given (31), it immediately follows that

Theorem 2 *When there are diminishing returns to R&D ($\beta < 1$) and the government does not intervene ($s_r = s_p = \tau = 0$),*
(i) both industry leaders and follower firms invest in R&D ($I_l > 0$ and $I_f > 0$),
(ii) follower firms are more innovative ($I_f > I_l$) when industry leaders have a relatively

small R&D cost advantage [$c_l^{\frac{1}{1-\beta}} > c_f^{\frac{1}{1-\beta}}(1-d^{\frac{-1}{\beta}})^{\frac{\beta}{1-\beta}}$], and

(ii) industry leaders are more innovative ($I_l > I_f$) when industry leaders have a relatively large R&D cost advantage [$c_l^{\frac{1}{1-\beta}} < c_f^{\frac{1}{1-\beta}}(1-d^{\frac{-1}{\beta}})^{\frac{\beta}{1-\beta}}$].

The general conclusion that emerges from studying both the constant and diminishing returns to R&D cases is that follower firms are more innovative than industry leaders unless leaders have a sufficiently large R&D cost advantage. When there are constant returns to R&D, this conclusion takes an extreme form because industry leaders do absolutely no research when they have a small R&D cost advantage. When there are diminishing returns to R&D, the model has more attractive properties: industry leaders invest in R&D activities but not as much as follower firms if their R&D cost advantage is small. According to Scherer [1984, ch. 11], Gelman Research Associates' large data base indicates that companies with over 10,000 employees are responsible for 35 percent of major innovations, whereas large firms in the Federal Trade Commission's (FTC) line-of-business survey account for 55 percent of the major innovations. Given Theorem 2, we interpret these numbers as indicating that the R&D cost advantage of the typical industry leader is neither large nor small, but of intermediate size.

4 Welfare

In this section, we explore the properties of the model when all allocation decisions are made by a social planner. We assume that the social planner's objective is to maximize the discounted utility of the representative consumer.

Since all intermediate inputs have the same marginal cost of production, clearly a social planner will only use the state-of-the-art quality intermediate inputs in each industry ω. Furthermore, given the state-of-the-art qualities $j(\omega,t)$ and the total resources $R(t)$ devoted to R&D at time t, if $Y(t)-X(t)$ is not maximized, then current consumption can be increased at no cost in terms of future consumption. Thus, using (2), the social planner maximizes

$$Y(t)-X(t)=\int_0^1 [AL^{1-\alpha}\lambda^{\alpha j(\omega,t)}X(\omega,t)^{\alpha}-X(\omega,t)]\,d\omega \tag{32}$$

with respect to $X(\omega, t)$ at each point in time t. Differentiating with respect to X inside the integral yields $X(\omega, t) = L(A\alpha)^{1/(1-\alpha)} d^{\frac{j(\omega,t)}{\beta}}$. Substituting this expression back into (32) and using the economy-wide resource constraint (16), we obtain a resource constraint relevant to the social planner:

$$Y(t) - X(t) = \Psi \cdot Q(t) = c(t)L + R(t), \tag{33}$$

where $\Psi \equiv (A\alpha)^{1/(1-\alpha)} L \left(\frac{1-\alpha}{\alpha}\right)$.

It is interesting to compare the resources devoted to intermediate input production by the social planner $X^w(t) = L(A\alpha)^{1/(1-\alpha)} Q(t)$ with the equilibrium outcome given by (7). The two levels coincide if and only if

$$s_p = 1 - \alpha, \tag{34}$$

that is, production of intermediate inputs is appropriately subsidized. By subsidizing production, the government can offset the distortion caused by monopoly pricing. With the appropriate production subsidy in place, (5) implies that intermediate inputs are sold to final good producers for exactly the marginal cost of production.

To determine the efficient allocations of R&D resources in each industry ω at each point in time t, the social planner maximizes $I(\omega, t) \equiv I_l(\omega, t) + \sum_i I_i(\omega, t)$ subject to $R_l(\omega, t) + \sum_i R_i(\omega, t) = \bar{R}$, (12) and (13). When $\beta < 1$, the planner will allocate R&D resources equally across followers in each industry. Given symmetric allocations, the instantaneous probability of follower success at any moment is defined by (14) and depends only on the total R&D resources allocated to followers in the industry. Furthermore industry resources are distributed between the leader and the followers so that

$$\frac{R_f(\omega, t)}{R_l(\omega, t)} = \frac{I_f(\omega, t)}{I_l(\omega, t)} = \Delta, \tag{35}$$

where $\Delta \equiv \left(\frac{c_l}{c_f}\right)^{\frac{1}{1-\beta}}$. In the limiting case where β=1, the planner will allocate all R&D resources to the leader given the leader's cost advantage in performing R&D $(c_f > c_l)$.

When $\beta < 1$, the equilibrium R&D resource allocation in (31) is the same as the planning resource allocation in (35) if and only if

$$\tau = d^{-\frac{1}{\beta}}. \tag{36}$$

By appropriately punishing follower firms that innovate, this turnover tax eliminates the R&D incentive differential between leader and follower firms. When $\beta = 1$, the equilibrium R&D resource allocation in (31) is the same as the planning resource allocation in (35) if and only if the turnover tax is prohibitively high [$\tau > 1 - \frac{c_f}{c_l}\left(\frac{d-1}{d}\right)$]. Then only efficient industry leaders engage in R&D activities.

The growth rate of the quality index $Q(t)$ depends on the total resources devoted to R&D and the allocation of these resources across firms. Having establishing the optimal allocation rule for distributing R&D resources across firms in each industry, the planner must choose the proper allocation of R&D resources across industries. $\dot{Q}(t) = \int_0^1 I(\omega,t)\left[d^{\frac{j(\omega,t)+1}{\beta}} - d^{\frac{j(\omega,t)}{\beta}}\right]d\omega$ given that $Q(t) \equiv \int_0^1 d^{\frac{j(\omega,t)}{\beta}} d\omega$. Substituting $I(w,t) = I_f(w,t) + I_l(w,t)$, (12) and (14) into this expression and then using (35) and $R(\omega,t) = R_l(\omega,t) + R_f(\omega,t)$ to simplify yields the rate of growth of the economy's quality index

$$\dot{Q}(t) = \delta(d^{\frac{1}{\beta}} - 1)\int_0^1 R(\omega,t)^{\beta} d^{\frac{(1-\beta)j(\omega,t)}{\beta}} d\omega, \tag{37}$$

where $\delta \equiv \left(\frac{1}{1+\Delta}\right)^{\beta}\frac{1}{c_l} + \left(\frac{\Delta}{1+\Delta}\right)^{\beta}\frac{1}{c_f}$. Because the economy's growth rate is an increasing function of δ, δ can be interpreted as a general measure of R&D efficiency. The efficient allocation of R&D resources across industries maximizes (37) subject to the resource constraint $\int_0^1 R(w,t)\,d\omega = R(t)$. The solution to this simple optimal control problem is that R&D resources are optimally distributed to each industry in proportion to the industry's relative quality factor, that is $\frac{R(t,w)}{R(t)} = \frac{d^{\frac{j(\omega,t)}{\beta}}}{Q(t)}$. This allocation implies that the instantaneous probability of R&D success is constant across industries and that

$$\dot{Q}(t) = \delta(d^{\frac{1}{\beta}} - 1)R(t)^{\beta}Q(t)^{1-\beta}. \tag{38}$$

Given (38), along the optimal balanced growth path for the economy, R&D resources as well as intermediate input use and output [see (33)], grow at the same rate as the economy's quality index. Then the resource constraint in (16) forces consumption to grow at this same rate.

We can now state the main optimal control problem that the social planner solves:

$$\max_{c(\cdot),R(\cdot)} \int_0^{\infty} \frac{c(t)^{1-\theta} - 1}{1-\theta} e^{-\rho t}\,dt$$

subject to the resource constraint $\Psi \cdot Q(t) = c(t)L + R(t)$, the state equation (38), the initial condition $Q(0) = 1$, and the control constraints $c(t) \geq 0$ and $R(t) \geq 0$ for all t. The Hamiltonian function for this problem is $H \equiv \frac{c^{1-\theta}-1}{1-\theta}e^{-\rho t} + \mu\delta(d^{\frac{1}{\beta}} - 1)R(t)^{\beta}Q(t)^{1-\beta} + \eta(\Psi Q - cL - R)$. Maximizing the Hamiltonian with respect to c and R yields the first order conditions for an interior solution $c^{-\theta}e^{-\rho t} = \eta(t)L = \mu(t)(d^{\frac{1}{\beta}} - 1)\delta\beta R(t)^{\beta-1}Q(t)^{1-\beta}L$. Differentiating with respect to time yields $-\frac{\dot{\mu}}{\mu} = \rho + \theta g$ where g is the constant growth rate in consumption, output, both R&D and intermediate input use, and the economy's quality index. Combining this with the costate equation $-\dot{\mu}(t) = \frac{\partial H}{\partial Q} = \eta(t)\Psi + \mu(t)(d^{\frac{1}{\beta}} - 1)\delta(1-\beta)R(t)^{\beta}Q(t)^{-\beta}$ and taking into account (38) and $\frac{\eta(t)}{\mu(t)} = (d^{\frac{1}{\beta}} - 1)\delta\beta R(t)^{\beta-1}Q(t)^{1-\beta}$, we obtain

$$F_o(g) \equiv \delta^{\frac{1}{\beta}}(d^{\frac{1}{\beta}} - 1)^{\frac{1}{\beta}}\beta\Psi g^{\frac{\beta-1}{\beta}} + [1 - \beta - \theta]g = \rho. \tag{39}$$

This equation implicitly defines the *optimal* balanced growth rate.

When $\theta \geq 1 - \beta$, the properties $F_o'(g) < 0$, $\lim_{g\to 0} F_o(g) = +\infty$, and $\lim_{g\to\infty} F_o(g) \leq 0$ imply that the function $F_o(g)$ and the constant function ρ intersect exactly once. Thus, the optimal growth rate is uniquely determined by (39).

When $\theta < 1 - \beta$, the properties $\lim_{g\to 0} F_o(g) = +\infty$, $\lim_{g\to 0} F_o'(0) = -\infty$, $F_o''(g) > 0$ and $\lim_{g\to\infty} F_o'(g) = 1 - \beta - \theta > 0$ imply that the function $F_o(g)$ is u-shaped. For sufficiently small ρ, $F_o(g) = \rho$ has no solution, implying that there is no balanced growth planning solution. We restrict attention in this paper to parameter values for which a balanced growth planning solution exists by assuming that $\rho > \inf_g F_o(g)$. Then the $F_o(g)$ function and the constant function ρ intersect twice, implying that there are two possible balanced growth planning solutions.

The higher growth intersection can be ruled out because it represents the growth rate that minimizes (instead of maximizes) discounted utility. To see why, consider the constant growth consumption path $c(t) = c_0e^{gt}$. Straightforward calculations using (10), (33) and (38) imply that $c_0 = (\Psi - [g/\{\delta(d^{\frac{1}{\beta}} - 1)\}]^{\frac{1}{\beta}})/L$. Substituting this consumption path into the consumer's utility function (10) and integrating yields $U = [\Psi - \{g/[\delta(d^{\frac{1}{\beta}} - 1)]\}^{\frac{1}{\beta}}]^{1-\theta}/[L^{1-\theta}(1-\theta)(\rho - g(1-\theta))] - 1/[(1-\theta)\rho]$. Differentiation of this function reveals that discounted utility is increasing in g when $F_o(g) > \rho$ and decreasing in g when $F_o(g) < \rho$. Thus, discounted utility is maximized at the lower-growth intersection and minimized at the higher-growth intersection. The optimal growth rate is uniquely determined and corresponds to the lower-growth intersection of the F_o and ρ functions.

If firm turnover is taxed at the optimal rate and production is subsidized at the optimal rate, then the equilibrium outcome will be welfare maximizing if the equilibrium growth rate coincides with the optimal growth rate. Using (23), (24), and (28) to substitute for k in (30) yields

$$F_e(g) \equiv \frac{\alpha}{1-s_r}\delta^{\frac{1}{\beta}}(d^{\frac{1}{\beta}}-1)^{\frac{1}{\beta}}\beta\Psi g^{\frac{\beta-1}{\beta}} + \left\{\frac{1-\beta}{1+\Delta} - \frac{\Delta}{(1+\Delta)(d^{\frac{1}{\beta}}-1)} - \theta\right\} g = \rho. \quad (40)$$

This equation implicitly defines the *equilibrium* balanced growth rate. By appropriately choosing the R&D subsidy rate s_r, the government can guarantee that the equilibrium growth rate is optimal.

In the limiting case where β converges to one and there are constant returns to R&D, the $F_e(g)$ and $F_o(g)$ functions are identical if and only if

$$s_r = 1 - \alpha = \frac{1}{E_d}, \quad (41)$$

where E_d is the elasticity of industry demand. Thus, (41) defines the optimal R&D subsidy rate, which is strictly positive. R&D subsidies are desirable because industry leaders cannot fully appropriate the benefits to society of their innovations, and even when their production is appropriately subsidized, their R&D incentives are insufficient.[7] Because this is a problem that results from a successful firm's inability to appropriate all the consumer surplus it creates, the R&D subsidy rate equals the inverse of the elasticity of industry demand. All else being equal, when demand is less elastic, the consumer surplus earned from buying new higher quality products is larger and it is optimal to subsidize R&D more.

It is optimal to subsidize R&D even more when there are diminishing returns to R&D ($\beta < 1$). To understand why, consider the extreme case where followers are unable to perform R&D, $c_f = +\infty$. In this case, the same R&D subsidy rate given in (41) solves the appropriation problem and the equilibrium growth rate is optimal. However, when followers are able to do R&D and are given the R&D subsidy in (41), R&D efforts are too

[7]It is straightforward to verify using the demand equation (4) that if leaders could perfectly price discriminate, then the market outcome would be Pareto efficient (without any production or R&D subsidies).

low in equilibrium since $F_e(g) < F_o(g) = \rho$ at the optimal growth rate g. Then a R&D subsidy rate greater than $1 - \alpha$ is needed for the equilibrium growth rate to be optimal. Therefore, it is not diminishing returns in the strict sense that causes the appropriation problem to get worse, it is the addition of follower firms to the R&D game. Follower participation reduces expected profit flows of industry leaders to a finite duration and this means that industry leaders capture an even smaller fraction of the social benefits they create with successful R&D efforts. To summarize, we have established

Theorem 3 *The welfare of the representative consumer is maximized and a Pareto efficient allocation of resources is attained when the government appropriately*
(i) subsidizes the production expenditures of industry leaders [$s_p = 1 - \alpha$],
(ii) subsidies the R&D expenditures of firms [$s_r = 1 - \alpha$ when $\beta = 1$ and $s_r > 1 - \alpha$ when $\beta < 1$], and
(iii) taxes the profits of new industry leaders [$\tau > 1 - \frac{c_f}{c_l}\left(\frac{d-1}{d}\right)$ when $\beta = 1$ and $\tau = d^{-1/\beta}$ when $\beta < 1$].

The properties established in Theorems 1, 2 and 3 are presented in a more compact form in Table 2. This table also presents corresponding properties derived in Barro and Sala-i-Martin [1995] for comparison purposes. Since Barro and Sala-i-Martin (abbreviated as B-SM in the table) solve for optimal consumption subsidies instead of optimal production subsidies, we present what their welfare results would have been if they had calculated optimal production subsidies instead.

5 Concluding Comments

A common property of many "quality ladders" R&D-driven growth models is that industry leaders do not engage in R&D activities. All of the R&D investment that drives economic growth is undertaken by industry followers in these models. This strong implication is not supported by actual firm behavior. In fact, it is commonplace for industry leaders to devote substantial resources to R&D activities. In this paper, we explore one possible reason why industry leaders often improve their own products.[8] We analyze the properties of a model where it is easier for industry leaders to improve their own products than for other firms.[9]

When there are constant returns to R&D effort and industry leaders have a small R&D cost advantage, we find that all the R&D investment that drives economic growth is done by follower firms in equilibrium. In this case, all innovations lead to firm turnover and we obtain Schumpeter's [1942] "process of creative destruction" in its purest form. However, as the R&D cost advantage of industry leaders is increased, eventually behavior switches and only industry leaders participate in R&D races. Then industry leadership positions are maintained forever and we obtain Schumpeter's [1928] "trustified capitalism" in its purest form.[10]

When there are diminishing returns to R&D effort, the model has more realistic properties: both industry leaders and follower firms engage in R&D activities. If the R&D cost advantage for leaders is large, then leaders discover the majority of new, higher quality products with followers occasionally innovating and becoming industry leaders. If the R&D cost advantage of industry leaders is small, then followers are responsible for a greater share of innovations and leadership turnover is more frequent.

The intuition behind these equilibrium properties is straightforward. When the leader R&D cost advantage is small, this advantage is more than offset by another disadvantage: leaders have less to gain from innovating than followers since they are already earning positive profits. Because R&D success by follower firms means stealing business away from

[8] Another possible reason is explored in Zolnierek [1997]: restricted entry into R&D races. When the number of firms that can participate in each R&D race is sufficiently small and each industry leader uses the same R&D technology as followers, Zolnierek shows that each industry leader will devote some resources to R&D activities; albeit, less resources than any other firm in the same industry (in the absence of government intervention).

[9] We do not study breakthrough research aimed at creating new industries (discovering new kinds of products that are not direct substitutes for existing products). Models where firms exclusively engage in breakthough research include Romer [1990] and Barro and Sala-i-Martin [1995, ch. 6].

[10] For empirical evidence concerning the importance of trustified capitalism, see Malerba and Orsenigo [1992]. Looking at thirty-three industries in four European countries, they found that two-thirds of the industries were better described by Schumpeter's characterization of trustified capitalism.

industry leaders, follower firms have stronger incentives to engage in R&D activities. Arrow [1962] recognized these differential R&D incentives when he showed that monopolists have weaker incentives to reduce their own costs than competitive firms. However, when the R&D cost advantage enjoyed by industry leaders is sufficiently large, then this advantage more than offsets the stronger revenue-based incentives followers have to do R&D. Then industry leaders are more innovative than industry followers.

The model also has some distinctive welfare properties. Discounted utility for the representative consumer is maximized when the government intervenes by appropriately subsidizing production, subsidizing R&D, and taxing firm turnover. We show that all three policy instruments are needed to achieve the first best outcome.

First, production subsidies for industry leaders are needed to offset the undesirable effects of monopoly pricing; namely, that industry leaders produce too little output from a social perspective. With the optimal production subsidy in place, the price charged by each industry leader equals the true marginal cost of production.

Second, R&D subsidies are needed to encourage industry leaders to devote more resources to R&D. Because industry leaders cannot perfectly price discriminate and instead practice standard monopoly pricing, they are not able to fully appropriate the consumer surplus gains generated by their innovations. As a result, equilibrium R&D investments by industry leaders are always too small in the absence of government intervention (even when industry leaders do most of the research in the economy). When there are constant returns to R&D effort, the optimal R&D subsidy rate only depends on the elasticity of industry demand, in fact, it equals the inverse of this elasticity. Thus, when demand is less elastic and consumers earn more consumer surplus, it is optimal to subsidize R&D at a higher rate. When there are diminishing returns to R&D, it is optimal for the government to subsidize R&D at an even higher rate than the inverse of the elasticity of industry demand.[11] With follower firms investing in R&D along the optimal path and sometimes displacing industry leaders by innovating, industry leaders have weaker incentives to invest in R&D than in the

[11] In a model where only followers do R&D, Stokey [1995] concludes that R&D taxes are more likely to be optimal when there are diminishing returns to R&D effort. In Stokey's model, the diminishing returns are external to the firms but internal to the industry.

constant returns case.

Third, turnover taxes are needed to discourage R&D effort by follower firms. This paper introduces a new reason for market failure that is not present in previous R&D-driven growth models: namely, that follower firms do relatively too much research. This source of market failure can be corrected by appropriately taxing the profits of new leaders (followers that become leaders by innovating). Because follower firms are not already earning positive economic profits, they have more to gain from innovating than leader firms. The optimal turnover tax offsets this R&D incentive differential. Without a turnover tax, industry leaders underachieve given their R&D abilities, industry followers overachieve given their R&D abilities and R&D resources are used inefficiently in the economy. Market forces generate too much "creative destruction" and this conclusion holds even when parameter values are such that almost all R&D is done by industry leaders.[12]

The above-mentioned equilibrium and welfare properties stand in sharp contrast with properties derived in the earlier literature. In contrast with our equilibrium results, Barro and Sala-i-Martin [1995] find that all R&D is done by industry leaders when they have the slightest R&D cost advantage. And in contrast with our welfare results, Aghion and Howitt [1992], Grossman and Helpman [1991], and Stokey [1995] show that either R&D taxes or R&D subsidies may be optimal, depending in complicated ways on various model parameters. Furthermore, in none of these three models are turnover taxes needed for welfare maximization since R&D resources are utilized efficiently when industry leaders do not participate in R&D races. We conclude that the equilibrium properties of "quality ladders" growth models become more realistic and the welfare properties significantly change when industry leaders have R&D cost advantages over follower firms and all firms face diminishing returns to R&D effort.

[12] In the constant returns to R&D case, the optimal turnover tax takes an extreme form: it is prohibitively high. By imposing a prohibitively high profits tax on follower firms that innovate, the government effectively discourages inefficient follower firms from investing any resources in R&D activities and sometimes becoming industry leaders. In the constant returns to R&D case, any leadership turnover is evidence that government policy is not optimally biased in favor of established firms and against new entrants (followers that become leaders by innovating).

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TABLE 1

1995 NET SALES AND R&D EXPENDITURES

OF SELECT INDUSTRY LEADERS*

Industry Leader	Net Sales	R&D Expenditure	R&D as a % of Sales
AT&T	$79.6	$3.7	4.7%
Boeing	$19.5	$1.3	6.5%
DuPont	$42.2	$1.1	2.5%
Eastman Kodak	$15.0	$0.9	6.2%
General Electric	$70.0	$1.9	2.7%
Hewlett Packard	$31.5	$2.3	7.3%
IBM	$71.9	$6.0	8.4%
Intel	$16.2	$1.3	8.0%
Johnson & Johnson	$18.8	$1.6	8.7%
Microsoft	$5.9	$0.9	14.5%
Motorola	$27.0	$2.2	8.1%
3M	$13.5	$0.9	6.6%
Xerox	$16.6	$1.0	5.7%

*In billions of dollars. Sources: Company annual reports published on the Internet.

TABLE 2

SUMMARY OF RESULTS

	B-SM (1995)	This paper	This paper
Returns to R&D expenditure	constant ($\beta = 1$)	constant ($\beta = 1$)	diminishing ($\beta < 1$)
Type of R&D equilibrium	Stackelberg	Nash	Nash
Equilibrium outcome when $c_f(d-1)/d < c_l$	$I_f = 0$, $I_l > 0$	$I_f > 0$, $I_l = 0$	$I_f > 0$, $I_l > 0$
Equilibrium outcome when $c_l < c_f(d-1)/d$	$I_f = 0$, $I_l > 0$	$I_f = 0$, $I_l > 0$	$I_f > 0$, $I_l > 0$
Optimal production subsidy	$s_p = 1/E_d$	$s_p = 1/E_d$	$s_p = 1/E_d$
Optimal R&D subsidy when $c_f(d-1)/d < c_l$	$s_r < 1/E_d$	$s_r = 1/E_d$	$s_r > 1/E_d$
Optimal R&D subsidy when $c_l < c_f(d-1)/d$	$s_r = 1/E_d$	$s_r = 1/E_d$	$s_r > 1/E_d$
Optimal turnover tax	$\tau = 0$	$\tau > 1 - (c_f/c_l)(d-1)/d$	$\tau = d^{-1/\beta}$