

# Endogenous Growth Without Scale Effects

Paul S. Segerstrom

*Department of Economics*

*Michigan State University*

*East Lansing, MI 48824*

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**Abstract:** This paper presents a simple R&D-driven endogenous growth model to shed light on some puzzling economic trends. The model can account for why patent statistics have been roughly constant even though R&D employment has risen sharply over the last 30 years. The model also illuminates why steadily increasing R&D effort has not lead to any upward trend in economic growth rates, as is predicted by earlier R&D-driven endogenous growth models with the “scale effect” property.

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# 1 Introduction

The number of scientists and engineers engaged in R&D in the United States has grown dramatically over time, from under 500,000 in 1965 to nearly 1 million in 1989. Other advanced countries have experienced even larger increases in R&D employment.<sup>1</sup> During the same time period, patent statistics have been roughly constant in many countries including the United States<sup>2</sup> and per capita growth rates have exhibited either a constant mean or have declined on average (see Jones [1995a]).

Increases in the resources devoted to R&D imply faster economic growth in many R&D-based endogenous growth models, including Romer [1990] and Segerstrom, et al. [1990]. Recently Jones [1995b] and Young [1995] have developed alternative models to explain why economic growth has not accelerated in spite of substantial increases over time in research effort. But neither model can account for the roughly constant rate of patenting. In Jones' model, for example, the number of patented innovations grows exponentially over time at the same rate as per capita consumption.

This paper presents a simple R&D-driven endogenous growth model that can account for the above-mentioned puzzling trends in R&D employment levels, patent statistics and economic growth rates. Instead of assuming that R&D workers become more productive over time in discovering new product designs, as in Romer [1990] and Jones [1995b], or that R&D productivity does not change over time, as in Grossman and Helpman [1991a] and Young [1995], we explore the theoretical implications of R&D becoming progressively more difficult over time. We assume that in each industry the most obvious ideas are discovered first, making it harder to find new ideas subsequently. This assumption is motivated by the experience of industries like the microprocessor industry. Describing the evolution of R&D difficulty in this industry, Malone [1985, p.253] writes, "But miracles, by definition, aren't easy and in the microprocessor business they get harder all the time. The challenges seem to grow with the complexity of the devices. . . that is, exponentially."

In each industry, firms engage in R&D in order to improve the quality of existing prod-

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<sup>1</sup>National Science Board [1993], Appendix table 3-22.

<sup>2</sup>WIPO [1983] and WIPO [various issues].

ucts, as in Segerstrom, et al. [1990] and Grossman and Helpman [1991a].<sup>3</sup> Firms that innovate and become industry leaders earn temporary monopoly profits as a reward for their R&D efforts. Due to positive growth in the population of consumers, the reward for innovating grows over time in each industry. However, counterbalancing this, innovating becomes progressively more difficult over time. The rate at which the economy grows is determined by the profit-maximizing R&D decisions of firms in each industry.

Regardless of initial conditions, equilibrium behavior in this model involves gradual convergence to a balanced growth path. Along this balanced growth path, economic growth proceeds at a constant rate and R&D employment grows at the same rate as total employment. Thus the model provides an explanation for why increased R&D employment over time has not led to any upward trend in economic growth. Also the instantaneous probability of an innovation in each industry is constant along the balanced growth path. Thus the model can account for the roughly constant rate of patenting in many countries since 1965.

Starting from the balanced growth path, a permanent unanticipated increase in the government subsidy to R&D investment leads to an increase in the rate of patenting. However, this normal effect is only temporary. With firms devoting more resources to research, R&D difficulty also increases more rapidly and the rate of technological change gradually falls back to the balanced growth rate. A permanently higher R&D subsidy permanently increases the fraction of workers doing research but only temporarily stimulates the rate of technological change. Like in Jones [1995b], we find that the long-run growth rate only depends on exogenous parameters, like the population growth rate.

Even though R&D subsidies do not have long-run growth effects, it does not follow

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<sup>3</sup>Scherer [1980,p.409] cites survey evidence that industrial firms devote 59% of their R&D efforts to the improvement of existing products, 28% to the development of new products and 13% to the development of new manufacturing processes. As Grossman and Helpman [1991a] point out, these data may understate the importance of vertical relative to horizontal differentiation in the innovation process since many new products replace old products that perform similar functions. Also the product improvements that we study are theoretically equivalent to cost reducing (process) innovations. An interesting topic for further research is to explore how the model's properties change when firms engage in two types of R&D: product improving and variety creating. In Jones [1995b] and Romer [1990], all R&D is directed at developing new horizontally differentiated products.

that a laissez faire public policy is welfare-maximizing. In general, the fraction of resources devoted to R&D along the equilibrium balanced growth path is not optimal (due to the presence of three external effects: the positive consumer surplus effect, the negative business stealing effect and the negative intertemporal R&D spillover effect). By subsidizing (taxing) R&D, the government can increase (decrease) the relative size of the R&D sector. We find that the larger is the size of innovations (and the corresponding markup of price over marginal cost charged by successful innovators), the smaller is the optimal R&D subsidy. Furthermore, when innovations are sufficiently large, the optimal R&D subsidy is negative: a R&D tax. In Jones' model, the reverse is the case: the larger is the size of innovations, the *larger* is the optimal R&D subsidy. In Grossman and Helpman [1991a], there is an n-shaped relationship between innovation size and the optimal R&D subsidy. Thus, these models have different welfare properties.

Simultaneously and independently of this paper, Kortum [1996] has also developed a R&D-driven growth model that can account for the same above-mentioned trends in R&D employment levels, patent statistics and economic growth rates. Kortum's paper complements this paper by showing that quite different assumptions about R&D processes can generate the same long-run implications. There are two significant differences between the two models. First, polar opposite assumptions are made about R&D directedness. All R&D activities are of a general unfocused nature in Kortum's model. An individual researcher is equally likely to discover a new technology in each industry (i.e., this researcher is just as likely to discover a new microprocessor design as a new drug for AIDS treatment). In contrast, all R&D activities are industry-specific and are focused on improving particular products in this paper (i.e., a researcher that is searching for a better microprocessor design does not accidentally discover a new drug for AIDS treatment). Second, polar opposite assumptions are made about R&D spillovers. In Kortum's model, the likelihood in an industry of finding new technologies that are better than some specified level depends on the average level of technological progress in other industries but not on the level of technological progress in the same industry. Kortum's model has strong across-industry R&D spillovers but no within-industry R&D spillovers. In this paper by contrast, the likelihood of finding technologies that are better than some specified level depends on how much

technological progress has occurred in the relevant industry but not on the levels of technological progress in other industries. Thus, this model has strong within-industry R&D spillovers but no across-industry R&D spillovers.<sup>4</sup>

The rest of this paper is organized as follows: In section 2, relevant empirical evidence about R&D and patenting behavior is reviewed and in section 3, the model is presented. Balanced growth equilibrium properties of the model are derived in section 4 and section 5 explores the welfare implications. Finally, section 6 offers some concluding comments.

## 2 Empirical Evidence

We begin by looking at the empirical evidence about aggregate R&D behavior. In Table 1, the total number of scientists and engineers engaged in R&D is reported for five advanced countries: the United States, Japan, West Germany, France and the United Kingdom.<sup>5</sup> What is striking about this data is the magnitude of the increase in R&D employment over time. During the time period from 1965 to 1989, R&D employment roughly doubled in the United States, tripled in West Germany and France, and quadrupled in Japan.

The same pattern emerges when we look at more disaggregated data. In Table 2, the R&D expenditures by United States, Japanese and West German manufacturers are reported (in millions of constant 1985 dollars, billions of constant 1985 yen and millions of constant 1985 deutsche marks, respectively).<sup>6</sup> This table shows that large increases in R&D

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<sup>4</sup>Because structural symmetry across industries is assumed and symmetric interventions are studied (the same R&D subsidy rate in all industries), all of the results derived in this paper continue to hold when R&D is unfocused. The assumption that R&D is focused becomes significant when there are differences between industries. For example, in Dinopoulos and Segerstrom [1996a], the model introduced in this paper is used to study the implications of trade barriers between countries. There it is shown that tariffs induce firms to exert more R&D effort in currently exporting industries and less R&D effort in currently importing industries (trade patterns change over time). In Kortum's model, it is not possible for firms to exert more R&D effort in some industries than in others. It is also worth noting that making R&D focused significantly changes the properties of Kortum's model.

<sup>5</sup>Source: National Science Board [1993].

<sup>6</sup>Source: National Science Board [1993].

Table 1: Scientists And Engineers Engaged In R&D (thousands)

	U. S.	Japan	W. G.	France	U. K.
1965	494.2	117.6	61.0	42.8	49.9
1975	527.4	225.2	103.7	65.3	80.5
1985	841.2	381.3	143.6	102.3	97.8
1989	949.3	461.6	176.4	120.7	NA

expenditures are not confined to a few industries but instead appear to be broad-based. The only manufacturers that did not substantially increase their R&D expenditures over the period from 1973 to 1990 were United States electrical machine manufacturers (and their Japanese and West German counterparts increased their R&D expenditures substantially).

If one takes seriously existing R&D-based models of endogenous growth, such as Romer [1990], and Segerstrom, et al. [1990], these increases in R&D expenditures should have lead to increases in (per capita) economic growth rates.<sup>7</sup> A key property of these early endogenous growth models is that larger economies devote more resources to R&D and as a result, grow faster. Yet, as is discussed at length in Jones [1995a], there has not been any upward trend in (per capita) economic growth rates in any of these countries during the relevant time period, in spite of large increases in population size.

In this paper, we explore one possible reason why increased R&D effort has not led to exploding growth, namely, that R&D has become progressively more difficult over time. Perhaps the best evidence that the productivity of R&D workers has dropped comes from the patent statistics.<sup>8</sup> As is noted by Kortum [1996], the number of patents per researcher

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<sup>7</sup>By “endogenous growth,” we mean that the rates of technological change and economic growth are endogenously determined based on the optimizing behavior of firms and consumers. The rate of technological change is exogenously given, for example, in Solow’s model of “exogenous growth” (see Barro and Sala-i-Martin [1995, chap.1]). A second definition of “endogenous growth” models is that they are models where public policies affect the long-run economic growth rate. The model in this paper satisfies the first definition but not the second.

<sup>8</sup>Pakes and Griliches [1984] and Hall, Griliches and Hausman [1986] use firm-level regressions of patents

Table 2: R&D Expenditures by Manufacturers

	United States		Japan		West Germany	
	1973	1990	1973	1990	1973	1990
Drugs & Medical	1,595	4,613	120	486	1,277	2,702
Office & Comp. Equip.	3,961	10,199	72	842	327	1,697
Electrical Machines	4,192	1,102	274	937	2,522	3,834
Radio, TV & Comm.	7,013	13,207	367	1,365	3,245	8,743
Aircraft	11,547	21,172	27	75	2,000	4,146
Scientific Instruments	2,197	5,013	57	316	428	771
Chemicals	3,241	5,441	324	847	4,027	7,646
Motor Vehicles	5,497	9,525	286	1,205	2,479	7,973
Other Manufacturing	7,694	9,673	780	2,301	3,612	9,236

has clearly declined in the United States during the last 40 years. Furthermore this decline is not a recent development or limited to the United States. Machlup [1962] shows that the patents per researcher ratio declined consistently from 1920 to 1960 in the U.S., and Even-son [1984] shows that the decline in patents per researcher is a world-wide phenomenon.<sup>9</sup> The decline in the number of patents per researcher suggests that patentable inventions have become increasingly difficult to discover.<sup>10</sup>

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on R&D to show that patents are indicators of inventive output. For a survey of the literature on patent statistics, see Griliches [1990].

<sup>9</sup>One possible explanation is that the decline is caused by shifts in industry composition. But Kortum [1993] finds that patenting relative to real R&D expenditure has fallen in all manufacturing industries. Another explanation is that the decline is due to a decreasing propensity of researchers to patent their inventions. But according to Mansfield [1986], researchers themselves do not report a decline in their propensity to patent their inventions.

<sup>10</sup>This conclusion is also reached by researchers studying specific industries: Baily and Chakrabarti [1985] for the textile and chemical industries, Henderson and Cockburn [1994] for the pharmaceutical industry and Malone [1985] for the microprocessor industry.

Although the patents per researcher ratio has fallen over time, the same is not true for the total number of patents granted. Figures 1 and 2 show the patents granted to residents in France, Germany, Japan, Sweden, the United States, and the United Kingdom. For the U.S., although there is considerable year to year fluctuations over time in the number of patents granted, it is remarkable how little the number of patents granted has changed over long stretches of time. The rate of patenting peaked in the 1960s, dropped in the 1970s and gradually recovered in the 1980s and early 1990s.<sup>11</sup> In spite of the recent increase, the number of patents granted in 1993 (53,200) was only slightly different from the number of patents granted in 1966 (54,600). The same pattern of a roughly constant patenting rate holds for France, Sweden and the United Kingdom. Japan is the only clear exception.<sup>12</sup> The number of patents granted to residents in Japan has increased steadily over time from 17,000 in 1965 to 77,000 in 1993. However, given that Japanese R&D employment was less than half of U.S. R&D employment during this period, these numbers indicate either that Japanese researchers were much more productive than their American counterparts, or more likely, that the Japanese patent office was unusually generous in granting patents for minor variations on other patented ideas (including ideas discovered in other countries). Thus, Japanese patent statistics may be less reliable indicators of inventive activity than the patent statistics of other countries.

In this paper, we present a R&D-driven endogenous growth model that is roughly consistent with the above mentioned empirical evidence. In particular, we construct a model where, along the balanced growth equilibrium path,

- aggregate R&D employment increases over time,
- industry-level R&D employment increases over time,
- the patents per researcher ratio decreases over time, and

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<sup>11</sup>The increase in patents grants in the 1980s may be the consequence of a major institutional change. As a result of the creation in 1983 of the Court of Appeals of the Federal Circuit, patents have received stronger protection in the U.S. (see Kortum [1996] and McConville [1994]).

<sup>12</sup>Germany experienced an upward trend in patenting in the 1980s but unfortunately data is not available for earlier periods.

- the total number of patents granted per year does not change over time.

The driving assumption behind these properties is that R&D becomes progressively more difficult over time.

In Young [1995], firms do two types of R&D: improve existing products and develop new varieties (industries). Young shows that the assumption that (quality improving) R&D becomes progressively more difficult over time is not needed to rule out scale effects. He assumes that quality improving R&D difficulty does not change over time and solves for an equilibrium where aggregate R&D employment increases over time without causing any upward trend in economic growth rates. However, Young's model can not explain *any* increases in R&D effort aimed at improving the quality of particular products (aggregate R&D employment grows entirely because the number of industries grows in his model). The model in this paper has properties that are more consistent with the experience of industries like the microprocessor industry, where R&D aimed at improving the quality of products has increased dramatically over time (see Malone [1995]). Also, Young's model predicts a constant patents per researcher ratio, which is inconsistent with the empirical evidence reviewed in this section.

### 3 The Model

#### 3.1 Some General Comments

We consider an economy with a continuum of industries indexed by  $\omega \in [0, 1]$ . In each industry  $\omega$ , firms are distinguished by the quality  $j$  of the products they produce. Higher values of  $j$  denote higher quality and  $j$  is restricted to taking on integer values. At time  $t = 0$ , the state-of-the-art quality product in each industry is  $j = 0$ , that is, some firm in each industry knows how to produce a  $j = 0$  quality product and no firm knows how to produce any higher quality product. To learn how to produce higher quality products, firms in each industry engage in R&D races. In general, when the state-of-the-art quality in an industry is  $j$ , the next winner of a R&D race becomes the sole producer of a  $j + 1$  quality product. Thus, over time, products improve as innovations push each industry up

its “quality ladder,” as in Grossman and Helpman [1991a].

### 3.2 Consumers and Workers

The economy has a fixed number of identical households that provide labor services in exchange for wages, and save by holding assets of firms engaged in R&D. Each individual member of a household is endowed with one unit of labor, which is inelastically supplied. The number of members in each family grows over time at the exogenous rate  $n > 0$ . We normalize the total number of individuals in the economy at time 0 to equal unity. Then the population of workers in the economy at time  $t$  is  $L(t) = e^{nt}$ . Each household is modelled as a dynastic family<sup>13</sup> which maximizes the discounted utility

$$U \equiv \int_0^\infty e^{nt} e^{-\rho t} \log u(t) dt \quad (1)$$

where  $\rho > n$  is the common subjective discount rate and  $u(t)$  is the utility per person at time  $t$ , which is given by

$$\log u(t) \equiv \int_0^1 \log \left[ \sum_j \lambda^j d(j, \omega, t) \right] d\omega. \quad (2)$$

In equation (2),  $d(j, \omega, t)$  denotes the quantity consumed of a product of quality  $j$  produced in industry  $\omega$  at time  $t$ , and  $\lambda > 1$  measures the size of quality improvements. Because  $\lambda^j$  is increasing in  $j$ , (2) captures in a simple way the idea that consumers prefer higher quality products.<sup>14</sup>

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<sup>13</sup>Barro and Sala-i-Martin [1995, chapter 2] provide more details on this formulation of the household’s behavior within the context of the Ramsey model of growth.

<sup>14</sup>Because innovations are proportional improvements in quality, the model has a balanced growth equilibrium. If innovations were less than proportional improvements in quality, growth would gradually fall to zero and if innovations were more than proportional improvements in quality, growth would gradually increase over time. We focus on the proportional improvements case both for tractability reasons (models with a balanced growth equilibrium are easier to analyze) and because economic growth rates appear to be roughly constant over time. As Jones [1997a] notes, if we tried to predict U.S. per capita GDP in 1994 by extrapolating a constant growth path using data from 1870 to 1929, the predicted value would only understate per capita GDP by 10.6 percent.

At each point in time  $t$ , each household allocates expenditure to maximize  $u(t)$  given the prevailing market prices. Solving this optimal control problem yields a unit elastic demand function ( $d = c/p$  where  $d$  is quantity demanded,  $c$  is per capita consumption expenditure and  $p$  is the relevant market price) for the product in each industry with the lowest quality adjusted price. The quantity demanded for all other products is zero. To break ties, we assume that when quality adjusted prices are the same for two products of different quality, a consumer only buys the higher quality product.

Given this static demand behavior, the intertemporal maximization problem of the representative household is equivalent to

$$\max_{c(t)} \int_0^{\infty} e^{-(\rho-n)t} \log c(t) dt \quad (3)$$

subject to the intertemporal budget constraint  $\dot{a}(t) = w(t) + r(t)a(t) - c(t) - na(t)$ , where  $a(t)$  denotes the per capita financial assets,  $w(t)$  is the wage income of the representative household member, and  $r(t)$  is the instantaneous rate of return. The solution to this intertemporal maximization problem obeys the well-known differential equation

$$\frac{\dot{c}(t)}{c(t)} = r(t) - \rho. \quad (4)$$

Thus, a constant per capita expenditure path is optimal when the market interest rate is  $\rho$ . A higher market interest rate induces consumers to save more now and spend more later, resulting in increasing per capita consumption over time.

### 3.3 Product Markets

In each industry, firms compete in prices. Labor is the only input in production and there are constant returns to scale. One unit of labor is required to produce one unit of output, regardless of quality. Labor markets are perfectly competitive and labor is used as the numeraire, so the wage of workers is set equal to one. Consequently, each firm has a constant marginal cost of production equal to unity.

To determine static Nash equilibrium prices and profits, consider any industry  $\omega \in [0, 1]$  where there is one quality leader and one follower firm (one step down in the quality ladder). This is the only type of industry configuration that occurs in equilibrium. With the

follower firm charging a price of one, the lowest price it can charge and not lose money, the quality leader earns the profit flow  $\pi(p) = (p - 1)c(t)L(t)/p$  from charging the price  $p$  if  $p \leq \lambda$ , and zero profits otherwise. These profits are maximized by choosing the limit price  $p = \lambda > 1$ . Thus the quality leader earns the profit flow

$$\pi^L \equiv \left( \frac{\lambda - 1}{\lambda} \right) c(t)L(t) \quad (5)$$

at time  $t$  and none of the other firms in the industry can do better than break even (by selling nothing at all).

### 3.4 R&D Races

Labor is the only input used to do R&D in any industry, is perfectly mobile across industries and between production and R&D activities. There is free entry into each R&D race and all firms in an industry have the same R&D technology. Any R&D firm  $i$  that hires  $\ell_i$  units of labor in industry  $\omega$  at time  $t$  is successful in discovering the next higher quality product with instantaneous probability  $A\ell_i/X(\omega, t)$ , where  $X(\omega, t)$  is a R&D difficulty index and  $A > 0$  is a given technology parameter. By instantaneous probability, we mean that  $\frac{A\ell_i}{X(\omega, t)} dt$  is the probability that the firm will innovate by time  $t + dt$  conditional on not having innovated by time  $t$ , where  $dt$  is an infinitesimal increment of time. The returns to engaging in R&D races are independently distributed across firms, across industries, and over time. Thus, the industry-wide instantaneous probability of innovative success at time  $t$  is simply

$$I(\omega, t) \equiv \frac{AL_I(\omega, t)}{X(\omega, t)} \quad (6)$$

where  $\sum_i \ell_i = L_I$  is the industry-wide employment of labor in R&D.

This R&D technology is new to the endogenous growth literature and the novel feature is the  $X(\omega, t)$  expression in the denominator of (6). We assume that R&D starts off being equally difficult in all industries [ $X(\omega, 0) = X_0$  for all  $\omega$ ,  $X_0 > 0$  given] and that R&D difficulty grows in each industry as firms do more R&D:

$$\frac{\dot{X}(\omega, t)}{X(\omega, t)} = \mu I(\omega, t), \quad (7)$$

where  $\mu > 0$  is exogenously given. With this formulation, we capture in a simple way the idea that as the economy grows and  $X(\omega, t)$  increases over time, innovating becomes

more difficult. The most obvious ideas are discovered first, making it harder to find new ideas subsequently.<sup>15</sup> (6) also implies that a constant innovation rate  $I$  can be consistent with positive growth in R&D labor  $L_I$  when  $X$  grows over time. Thus, this model has the potential to explain the evidence cited at the beginning of the paper: growth in the number of scientists and engineers engaged in R&D without accelerating economic growth.

Let  $v(\omega, t)$  denote the expected discounted profit or reward for winning a R&D race (in industry  $\omega$  at time  $t$ ) and  $s_R$  denote the fraction of each firm's R&D costs paid by the government. We will assume that the government finances the chosen R&D subsidy  $s_R$  using lump-sum taxation. Then at each point in time  $t$ , each R&D firm  $i$  chooses its labor input  $\ell_i$  to maximize its expected profits  $\frac{v(\omega, t)A\ell_i}{X(\omega, t)} - \ell_i(1 - s_R)$ . If  $v(\omega, t) > X(\omega, t)(1 - s_R)/A$ , then  $\ell_i = +\infty$  is profit maximizing and if  $v(\omega, t) < X(\omega, t)(1 - s_R)/A$ , then  $\ell_i = 0$  is profit maximizing. Only when

$$v(\omega, t) = \frac{X(\omega, t)(1 - s_R)}{A} \quad (8)$$

is it profit maximizing for firms to devote a positive (finite) amount of labor to R&D. (8) tells us that when R&D is more difficult [ $X(\omega, t)$  is higher] or when the government subsidizes R&D less [ $s_R$  is lower], the reward for winning a R&D race must be larger to induce positive R&D effort. When (8) holds, firms are globally indifferent concerning their choice of R&D effort. Given the symmetric structure of the model, we focus on equilibrium behavior where the R&D intensity  $I(\omega, t)$  is the same in all industries  $\omega$  at time  $t$  and is strictly positive. Thus the  $\omega$  argument of functions is dropped in the remaining analysis.

The stock market valuation of monopoly profits provides another equilibrium condition that relates the expected discounted profits to the flow of profits and the instantaneous interest rate.<sup>16</sup> Over a time interval  $dt$ , the shareholder receives a dividend  $\pi^L(t) dt$ , and the

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<sup>15</sup>Although we do not explicitly model the process that leads to increasing R&D difficulty, we think of firms as choosing among an infinite array of R&D projects with varying degrees of expected difficulty. Firms start off exploring the ex ante most promising projects and when success does not materialize, they gradually switch to ex ante less promising projects. We view (7) as representing a convenient reduced form description of this type of R&D search process.

<sup>16</sup>There exists a stock market in the economy that channels consumer savings to firms engaged in R&D and helps households to diversify the risk of holding stocks issued by R&D firms.

value of the monopolist appreciates by  $\dot{v}(t) dt$  in each industry. Because each quality leader is targeted by other firms that conduct R&D to discover the next higher quality product, the shareholder suffers a loss of  $v(t)$  if further innovation occurs. This event occurs with probability  $I(t) dt$ , whereas no innovation occurs with probability  $[1 - I(t)] dt$ . Efficiency in financial markets requires that the expected rate of return from holding a stock of a quality leader is equal to the riskless rate of return  $r(t) dt$  that can be obtained through complete diversification:  $\frac{\pi^L}{v} dt + \frac{\dot{v}}{v}(1 - I) dt - \left[\frac{v-0}{v}\right] I dt = r dt$ . Taking limits as  $dt$  approaches zero, we obtain:

$$v(t) = \frac{\pi^L(t)}{r(t) + I(t) - \frac{\dot{v}(t)}{v(t)}}. \quad (9)$$

The profits earned by each leader  $\pi^L$  are appropriately discounted using the interest rate  $r$  and the instantaneous probability  $I$  of being driven out of business by further innovation. Also taken into account in (9) is the possibility that these discounted profits grow over time. R&D profit maximization implies that the discounted marginal revenue product of an idea (9) must equal its marginal cost (8) at each point in time, that is,

$$\frac{x(t)(1 - s_R)}{A} = \frac{\left(\frac{\lambda-1}{\lambda}\right) c(t)}{r(t) + (1 - \mu)I(t)}, \quad (10)$$

where  $x(t) \equiv \frac{X(t)}{L(t)}$ . Note that (8) implies that  $\frac{\dot{v}(t)}{v(t)} = \frac{\dot{X}(t)}{X(t)} = \mu I(t)$ .

### 3.5 The Labor Market

In each industry  $\omega$  at time  $t$ , consumers only buy from the current quality leader and pay the equilibrium price  $\lambda$ . Since consumer demand is unit elastic,  $c(t)L(t)/\lambda$  workers must be employed by the current quality leader to produce enough to meet consumer demand. In addition,  $L_I(t)$  workers are employed by R&D firms in each industry at time  $t$ . With a measure one of industries, full employment of workers is satisfied when  $L(t) = \frac{c(t)L(t)}{\lambda} + L_I(t)$  holds for all  $t$ . Substituting using  $x(t) \equiv \frac{X(t)}{L(t)}$  and  $L_I(t) = I(t)X(t)/A$  into the full employment condition, we obtain

$$1 = \frac{c(t)}{\lambda} + \frac{I(t)x(t)}{A}. \quad (11)$$

At any point in time  $t$ , more consumption  $c(t)$  comes at the expense of less R&D investment  $I(t)$ .

### 3.6 Balanced Growth Equilibria

We now solve the model for balanced growth equilibrium paths where all endogenous variables grow at constant (not necessarily the same) rates and firms invest in R&D [ $I(t) > 0$  for all  $t$ ].

Given  $\mu > 0$ , (7) implies that  $I$  must be a constant over time. It then follows from (11) that  $c$  and  $x$  must also be constants. Thus, any balanced growth equilibrium must involve  $c$ ,  $x$  and  $I$  taking on constant values over time.

Differentiating (6) with respect to time using (7) yields  $\frac{\dot{I}}{I} = \frac{\dot{L}_I}{L_I} - \mu I = 0$ . (6) and (11) together imply that in any balanced growth equilibrium, employment in the R&D sector  $L_I(t)$  must grow at the same rate as the population ( $n$ ). Thus, there is a unique balanced growth R&D intensity

$$I = \frac{n}{\mu}. \quad (12)$$

The level of R&D investment is completely determined by the exogenous rate of population growth  $n > 0$  and the R&D difficulty growth parameter  $\mu > 0$ . The balanced growth innovation rate is higher when the population of consumers grows more rapidly or when R&D difficulty increases more slowly over time. Note that if there is no increase in R&D difficulty over time ( $\mu \leq 0$ ), then there is no balanced growth equilibrium. Instead, the growth rate of the economy increases without bound over time.<sup>17</sup>

With  $I$  given by (12), and (4) implying that the equilibrium interest rate is  $r(t) = \rho$ , (10) yields a balanced growth R&D condition

$$\frac{(1 - s_R)x}{A} = \frac{\left(\frac{\lambda-1}{\lambda}\right)c}{\rho + \frac{n}{\mu} - n}, \quad (13)$$

and (11) yields a balanced growth resource condition

$$1 = \frac{c}{\lambda} + \frac{nx}{A\mu}. \quad (14)$$

Both balanced growth conditions are illustrated in Figure 3. The vertical axis measures

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<sup>17</sup>The parameter  $n$  should be interpreted as the world population growth rate, not as the population growth rate of any particular country. In the model,  $n$  represents the growth rate of consumers that industry leaders sell to and in a world with international trade, firms are not limited to only selling to domestic consumers.

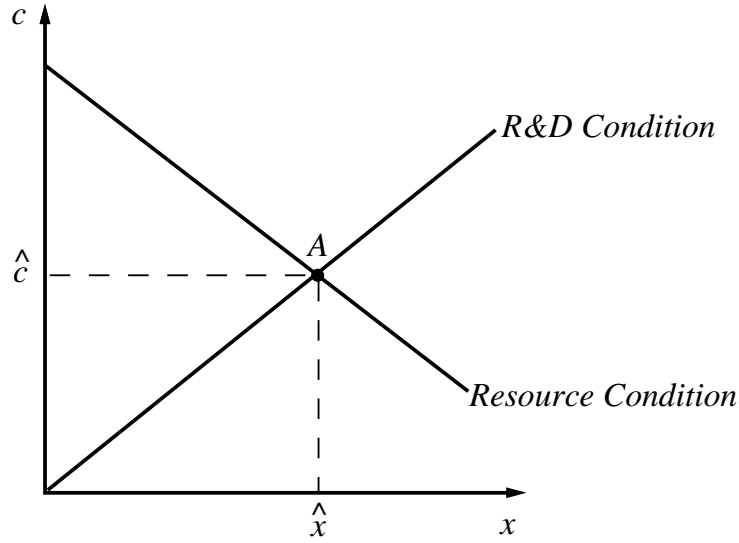


Figure 3: The unique balanced growth equilibrium

consumption per capita  $c$  and the horizontal axis measures relative R&D difficulty  $x$ . The R&D condition is upward sloping in  $(x, c)$  space, indicating that when R&D is relatively more difficult, consumer expenditure must be higher to justify positive R&D effort by firms. The resource condition is downward sloping in  $(x, c)$  space, indicating that when R&D is relatively more difficult and more resources are used in the R&D sector to maintain the balanced growth innovation rate  $I$ , less resources are available to produce goods for consumers, so individual consumers must buy less. The unique intersection between the R&D and resource conditions at point  $A$  determines the balanced growth values of consumption per capita  $\hat{c}$  and relative R&D difficulty  $\hat{x}$ .

If  $x = \hat{x}$  at time  $t = 0$ , then an immediate jump to the balanced growth path can occur. Otherwise, it is imperative to investigate the transitional dynamic properties of the model.

Differentiating relative R&D difficulty  $x(t) \equiv \frac{X(t)}{L(t)}$  with respect to time using (7) yields  $\frac{\dot{x}(t)}{x(t)} = \mu I(t) - n$ . Substituting into this expression for  $I(t)$  using the resource condition (11) yields one differential equation that must be satisfied along any equilibrium path for the economy:

$$\dot{x}(t) = \mu A \left( 1 - \frac{c(t)}{\lambda} \right) - nx(t) \quad (15)$$

Since the RHS of (15) is decreasing in both  $x$  and  $c$ ,  $\dot{x}(t) = 0$  defines the downward-sloping curve in Figure 4. Starting from any point on this curve, an increase in  $x$  leads to  $\dot{x} < 0$

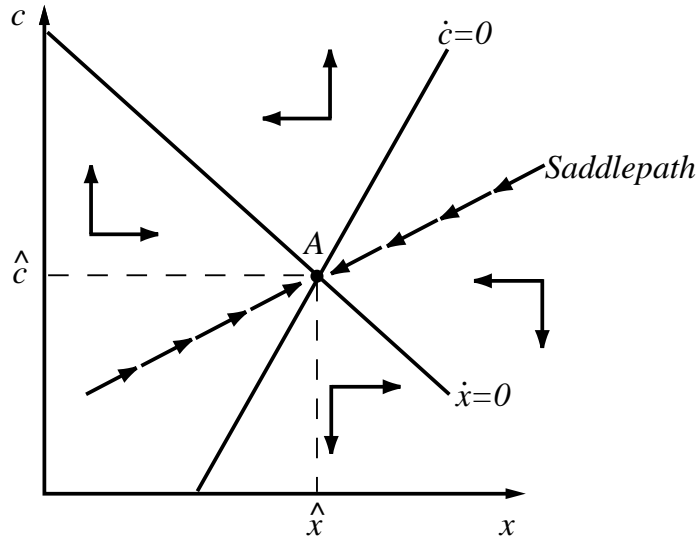


Figure 4: Stability of the balanced growth equilibrium

and a decrease in  $x$  leads to  $\dot{x} > 0$ , as is illustrated by the horizontal arrows in Figure 4. Solving (11) for  $I(t)$ , (10) for  $r(t)$  and then substituting into (4) yields a second differential equation that must be satisfied along any equilibrium path for the economy:

$$\dot{c}(t) = c(t) \left[ \frac{(\lambda - 1)A}{\lambda(1 - s_r)} \frac{c(t)}{x(t)} + \frac{(\mu - 1)A}{x(t)} \left( 1 - \frac{c(t)}{\lambda} \right) - \rho \right] \quad (16)$$

If  $\mu \leq 1$ , then the  $\dot{c}(t) = 0$  curve is definitely upward sloping in  $(x, c)$  space. Starting from any point on this curve, an increase in  $x$  leads to  $\dot{c} < 0$  and a decrease in  $x$  leads to  $\dot{c} > 0$ , implying that there exists an upward-sloping saddlepath. If  $\mu$  is slightly greater than 1, then the  $\dot{c}(t) = 0$  curve is still upward sloping in  $(x, c)$  space and there exists an upward sloping saddlepath (this case is illustrated in Figure 4). Even if  $\mu$  is significantly greater than 1 and the  $\dot{c}(t) = 0$  curve is downward sloping, there still exists an upward-sloping saddlepath going through the unique balanced growth equilibrium point  $A$ . Thus the balanced growth equilibrium is saddlepath stable. By jumping onto this saddlepath and staying on it forever, convergence to the balanced growth equilibrium occurs, just like in the neoclassical growth model.<sup>18</sup>

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<sup>18</sup>We are able to solve for equilibrium transition paths because the model is simpler than many other endogenous growth models. For example, to analyze out-of-steady-state behavior, Jones [1995b, p.773] introduces the ad hoc assumption of a constant (and exogenously given) R&D employment ratio over time. We can solve for how this ratio evolves over time. Equation (11) implies that the R&D employment ratio

Along a balanced growth path, we can determine the fraction of the labor force devoted to R&D. (6) implies that  $\frac{L_I(t)}{L(t)} = \frac{Ix}{A}$ . Solving (13) and (14) for  $x$  yields a final balanced growth condition:

$$\frac{L_I}{L} = \frac{1}{1 + \frac{(1-s_R)}{(\lambda-1)} \left[1 + \frac{\rho-n}{I}\right]} \quad (17)$$

Given (12), the balanced growth fraction of the labor force devoted to R&D  $\frac{L_I}{L}$  is completely determined by parameter values. Interestingly, although a higher R&D subsidy has no effect on the long-run innovation rate, it does increase the fraction of workers in the economy doing R&D.

Along a balanced growth path, the representative consumer's utility grows at a constant rate. To solve for this utility growth rate, we substitute the static consumer demand for quality leader products  $d(j, \omega, t) = c(t)/\lambda$  into (2) to obtain

$$\log u(t) = \log \frac{c(t)}{\lambda} + \int_0^1 \log \lambda^{j(\omega,t)} d\omega = \log \frac{c(t)}{\lambda} + \Phi(t) \log \lambda \quad (18)$$

where  $\Phi(t) \equiv \int_0^t I(\tau) d\tau$  is the expected number of R&D successes in the typical industry before time  $t$ . Taking into account that  $c(t)$  and  $I(t)$  are constants along a balanced growth path, differentiation of (18) yields

$$\frac{\dot{u}}{u} = I \log \lambda = \frac{n}{\mu} \log \lambda. \quad (19)$$

Individual utility growth depends on the rate at which new higher quality products are introduced. Treating individual utility growth as our measure of long-run economic growth for the economy, (19) implies that a higher R&D subsidy has no effect on long-run economic growth since it does not impact the long-run innovation rate in any industry.<sup>19</sup>

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along an equilibrium transition path is given by  $\frac{L_I(t)}{L(t)} = 1 - \frac{c(t)}{\lambda}$ . If the economy starts off to the left of point  $A$  in Figure 4, then consumption per capita  $c$  rises and the R&D ratio falls over time. The reverse holds if the economy starts off to the right of point  $A$ . Thus, the share of labor devoted to R&D is not constant along any equilibrium transition path.

<sup>19</sup>Romer [1986] reports evidence of an upward trend in the economic growth rates of leading countries from 1700 to 1979. Equation (19) provides one possible explanation for this trend because there was also an upward trend in  $n$  during this period of time (see Kremer [1993]). Since the economic growth rate is an increasing function of the population growth rate, it stands to reason that an upward trend in the population growth rate would lead to an upward trend in the economic growth rate.

In solving for the balanced growth equilibrium, we have let the wage rate for labor be the numeraire ( $w(t) = 1$  for all  $t$ ). Thus we have implicitly assumed that the nominal wage is constant over time. Since quality-adjusted prices are falling over time due to innovation, the real wage  $W(t)$  must be rising and we can calculate the long-run equilibrium growth rate of the real wage  $g$ . In the typical industry, an innovation occurs every  $1/I$  years and leads to a proportional real wage increase of  $\lambda$  (the innovation size). It follows that  $W(\frac{1}{I}) = \lambda W(0) = W(0)e^{g \cdot 1/I}$  and solving for  $g$  yields  $g = I \log \lambda$ . Thus the utility growth rate is also the real wage growth rate in this model.<sup>20</sup>

In a balanced growth equilibrium, the real value of patented innovations rises over time ( $\frac{\dot{v}(t)}{v(t)} + g = \mu I + g = n + g$ ) causing firms to expend ever greater resources to discover them ( $\frac{\dot{L}_I(t)}{L_I(t)} = n$ ). Although the patents per researcher ratio ( $\frac{A}{X(t)}$ ) falls over time, this is more than offset by the increase in the real value of patented innovations discovered per researcher, so the real wage of researchers (and workers in general) grows over time at the rate  $g$ .<sup>21</sup>

As we documented in Table 1, the number of scientists and engineers engaged in R&D has increased dramatically over time. In fact, as is illustrated in Figure 5, this number has grown over time at an even faster rate than the total labor force, indicating that we are not

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<sup>20</sup>The R&D difficulty parameter  $\mu$  plays a key role in the analysis. Using data on the world population growth rate  $n$ , the average percentage markup of price over marginal cost (a proxy for  $\lambda - 1$ ) and the average per capita GDP growth rate (a proxy for  $g$ ), this parameter can be empirically estimated using (19). Although we assume structural symmetry across industries throughout the paper, the R&D difficulty parameter probably varies considerably across industries. A more direct way of empirically estimating  $\mu$  at the industry level would be to use data on the declining patent per researcher ratio to estimate  $\dot{X}(t)/X(t)$ , use the rate of patenting as a proxy for  $I$  and then infer  $\mu$  using (7).

<sup>21</sup>The implication of the model that individual patent values increase over time is difficult to verify empirically since patent rights are seldom marketed and their private value is in general unobserved. Schankerman and Pakes [1986] attempt to indirectly infer patent values from the renewal decisions of patent holders. Consistent with the model, they estimate that from 1955 to 1975, the value of individual patents increased by 40%, 86% and 93% in the United Kingdom, France and Germany, respectively. They also find that for the shorter period 1965-1975, patent values per scientist and engineer were roughly constant, whereas the model predicts positive growth. However, during the relevant time period, R&D employment grew at a much faster rate than total employment in all three countries.

observing balanced growth equilibrium behavior.<sup>22</sup> Throughout the relevant time period (1965-1989), the United States had the highest fraction of workers engaged in R&D. In all the other countries, the share of labor devoted to R&D has steadily increased over time, with the biggest increase occurring in Japan. Part of the increase in these countries can be attributed to convergence to American levels but even the share of labor devoted to R&D in the United States has increased, particularly after 1975.

The model provides one explanation for why the share of labor devoted to R&D could increase over time, namely, an increase in the R&D subsidy rate. Equation (17) implies that a permanent increase in  $s_R$  permanently increases  $\frac{L_I}{L}$ . However a more likely explanation is developed in Dinopoulos and Segerstrom [1996b], where a two-country version of the model is analyzed and it is shown that permanently lower tariff barriers cause the share of labor devoted to R&D to permanently increase in both trading countries. The international effort to cut tariff and nontariff barriers embodied in GATT, NAFTA, WTO and other agreements has contributed to an explosion in international trade. In the United States, imports and exports were about 3 percent of GDP in 1970, as opposed to 10-12 percent today. According to World Bank figures, between 1965 and 1990, the share of output exported rose for low-income countries from 8 to 18 percent, for middle-income countries from 17 to 25 percent and for high-income countries, from 12 to 20 percent.<sup>23</sup> Increased integration of global markets may be responsible for much of the increase in the R&D employment ratio over time.<sup>24</sup>

## 4 Optimal Growth

In this section, we explore the balanced growth properties of the model when all allocation decisions are made by a social planner. We assume that the social planner's objective is to

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<sup>22</sup>Source: National Science Board [1993]. The fraction of the labor force engaged in R&D cannot increase forever. When it levels out, Jones [1997a] predicts that there will be a significant drop in the U.S. per capita GDP growth rate.

<sup>23</sup>See Richardson [1995].

<sup>24</sup>Jones [1997a] reaches the same conclusion based on an analysis of U.S. time-series data.

maximize the discounted utility of the representative family.

We will first show that the social planner chooses the same R&D intensity in each industry. Allowing for different R&D intensities in different industries, the labor market constraint is  $L(t) = \frac{c(t)L(t)}{\lambda} + \int_0^1 L_I(\omega, t) d\omega$ . Thus the quantity consumed of each state-of-the-art quality product by the representative consumer at time  $t$  is  $\frac{c(t)}{\lambda} = 1 - \int_0^1 \frac{L_I(\omega, t)}{L(t)} d\omega$ . For given resources devoted to R&D at time  $t$ ,  $\int_0^1 L_I(\omega, t) d\omega$ , clearly the social planner wants to choose the distribution of R&D expenditures across industries to maximize  $\frac{d}{dt} \int_0^1 \log \lambda^{j(\omega, t)} d\omega = \log \lambda \int_0^1 I(\omega, t) d\omega = \log \lambda \int_0^1 \frac{AL_I(\omega, t)}{X(\omega, t)} d\omega$  [where we have used (6)]. This implies that at any point in time  $t$ , all R&D is done in those industries with the lowest  $X(\omega, t)$  and with the social planner carrying out such a policy throughout time,  $X(\omega, t) = X(t)$  and  $I(\omega, t) = I(t)$  for all  $\omega$  and  $t$ . With the R&D intensity not varying across industries,  $\dot{\Phi}(t) = I(t)$  and (7) then imply that

$$X(t) = X_0 e^{\mu \Phi(t)}. \quad (20)$$

Substituting the appropriately simplified expression  $\frac{c(t)}{\lambda} = 1 - \frac{X(t)I(t)}{AL(t)}$  back into the discounted utility function (1) using (20) and  $L(t) = e^{nt}$ , we can solve for the discounted utility of the representative family. The optimal control problem facing the social planner is given by

$$\max_{I(\cdot)} \int_0^\infty e^{-(\rho-n)t} \left\{ \Phi(t) \log \lambda + \log \left[ 1 - \frac{I(t)X_0 e^{\mu \Phi(t)}}{Ae^{nt}} \right] \right\} dt \quad (21)$$

subject to the state equation  $\dot{\Phi}(t) = I(t)$ , the initial conditions  $[\Phi(0) = 0]$ , and the control constraints  $[\frac{A}{x(t)} \geq I(t) \geq 0$  for all  $t$ ].

This optimal control problem is solved in Appendix A. We find that there is a unique balanced growth solution: the innovation rate is

$$I = \frac{n}{\mu}, \quad (22)$$

and the R&D employment ratio is

$$\frac{L_I}{L} = \frac{1}{1 + \frac{\rho}{I \log \lambda}}. \quad (23)$$

Even though the optimal innovation rate (22) coincides with the equilibrium innovation rate (12), the government can in general improve welfare by intervening in the economy.

Comparing (23) with (17), the optimal R&D subsidy rate  $s_R$  must satisfy

$$(1 - s_R) \frac{\log \lambda}{\rho - n} = \left[ \frac{\lambda - 1}{I + \rho - n} \right] \left[ 1 + \frac{n}{\rho - n} \right]. \quad (24)$$

Since  $\frac{\lambda-1}{\log \lambda}$  is a globally increasing function of  $\lambda$  and the equilibrium innovation rate  $I$  given by (12) does not depend on  $\lambda$ , (24) implies that the optimal R&D subsidy rate  $s_R$  is a globally decreasing function of  $\lambda$ , holding all other parameter values fixed.<sup>25</sup> The optimal R&D subsidy  $s_R$  is also a globally decreasing function of  $\mu$ , holding all other parameter values fixed. Since  $\lim_{\lambda \rightarrow \infty} \frac{\log \lambda}{\lambda-1} = 0$ , it is optimal to tax R&D ( $s_R < 0$ ) if  $\lambda$  is sufficiently large. Furthermore, (24) implies that it is optimal to subsidize R&D ( $s_R > 0$ ) if  $\mu > 0$  is sufficiently small. Thus, both R&D subsidies and R&D taxes can be optimal, depending on the parameters of the model.

To understand the intuition behind these welfare results, it is helpful to think about three external effects of the marginal innovation that are present in the model (for formal derivations of these external effects, see Appendix B).

First, every time a firm innovates, consumers benefit because they can buy a higher quality product at the same price that they used to pay for a lower quality substitute. Furthermore these consumer benefits last forever because future innovations build on all the innovations of the past. We refer to this positive external effect of the marginal innovation as the *consumer surplus effect*. Because individual R&D firms do not take into account the external benefits to consumers that innovations generate in their profit-maximization calculations, this external effect represents one reason why firms may under-invest in R&D activities from a social perspective. The consumer surplus effect measures  $\frac{\log \lambda}{\rho - n}$  in terms of the utility metric given by (1). Consumers benefit more from the marginal innovation when innovations represent larger improvements in product quality (large  $\lambda$ ), consumers are more patient (small  $\rho$ ), and there are more consumers in the future to benefit from the marginal innovation (large  $n$ ).

Second, every time a firm innovates, it drives another firm out of business. The dis-

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<sup>25</sup>The proof of the first claim is as follows: First we note that  $\lambda \cdot \log \lambda > \lambda - 1$  for all  $\lambda > 1$  since  $(\lambda \cdot \log \lambda)' = 1 + \log \lambda > (\lambda - 1)' = 1$  and equality holds when  $\lambda = 1$ . Therefore, using the fact that  $\log \lambda > \frac{\lambda-1}{\lambda}$  holds for all  $\lambda > 1$ , differentiation reveals that  $\frac{\lambda-1}{\log \lambda}$  is an increasing function of  $\lambda$  for all  $\lambda > 1$ .

placed firm forfeits a stream of monopoly profits and the owners of the displaced firm experience a windfall loss. By itself,<sup>26</sup> this loss in profit income contributes to lower aggregate consumer expenditure and hence, lower profits for other quality leader firms. We refer to this negative external effect of the marginal innovation as the *business stealing effect*.<sup>27</sup> Because individual R&D firms do not take into account in their profit-maximization calculations the external losses incurred by other firms from the marginal innovation, this external effect represents one reason why firms may over-invest in R&D activities from a social perspective. The business stealing effect measures  $\frac{\lambda-1}{\rho+I-n}$  in terms of the utility metric given by (1). The business stealing effect is larger when firms earn higher per unit profit margins ( $\lambda - 1$  large), there are more consumers in the future that firms can sell to ( $n$  large), future profits are lightly discounted ( $\rho$  small) and industry leader firms expect to be in business for a long time ( $I$  small).

Third, a firm that innovates is a firm that is investing in R&D and this R&D investment raises the R&D costs of other firms in the future. R&D investment today permanently increases R&D difficulty, which means that more resources need to be devoted to R&D activities in the future to maintain the constant balanced growth innovation rate and less resources are available for producing consumer goods. Furthermore, since lower production of consumer goods must be associated with lower consumer expenditure, there is also a multiplier effect from the increase in R&D difficulty, as lower consumer expenditure leads to lower aggregate profits, which leads to still lower consumer income and expenditure. We refer to this negative external effect of the marginal innovation as the *intertemporal R&D spillover effect*. Because individual R&D firms do not take into account in their profit-maximization calculations the increased R&D costs incurred by other firms from the marginal innovation, this external effect represents a second reason why firms may over-invest in R&D activities from a social perspective. The intertemporal R&D spillover effect measures  $\frac{\lambda-1}{\rho+I-n} \frac{n}{\rho-n}$  in terms of the utility metric given by (1). The intertemporal R&D

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<sup>26</sup>That is, ignoring the increased profits earned by the new quality leader.

<sup>27</sup>When calculating the size of the business stealing effect in Appendix B, we take into account that the lower profits for quality leader firms contribute to further decreases in consumer income, consumer expenditure and firm profits.

spillover effect is larger when innovations generate bigger increases in R&D difficulty ( $\mu$  large or  $I = n/\mu$  small), the relative size of the R&D sector increases ( $\lambda$  large), and consumers attach more importance to the future ( $\rho$  small).<sup>28</sup>

Equation (24) reveals that an increase in the innovation size parameter  $\lambda$  increases the size of all three external effects. When innovations are larger, they are associated with bigger decreases in quality-adjusted prices (a larger consumer surplus effect), innovative firms earn higher profit flows (a larger business stealing effect) and these higher profits in turn lead to more R&D investment (a larger intertemporal R&D spillover effect). However, given our assumption of diminishing marginal utility in consumption ( $\log \lambda$  is a strictly concave function of  $\lambda$ ), as  $\lambda$  increases, the consumer surplus effect increase is dominated by increases in the two negative external effects. Thus, as  $\lambda$  increases, so also does the market bias toward over-investment in R&D and the optimal R&D subsidy rate  $s_R$  is a decreasing function of the innovation size parameter  $\lambda$ .

An increase in  $\mu$  does not lead to any change in the consumer surplus effect but increases both the business stealing and intertemporal R&D spillover effects. When the R&D difficulty parameter  $\mu$  increases, this leads to a decrease in the equilibrium innovation rate  $I = n/\mu$  in each industry. Since innovative firms now expect to earn profits for a longer period of time before being driven out of business, the business stealing effect is larger and since a larger reward for R&D success leads to more R&D investment, the intertemporal R&D spillover effect is larger as well. Thus, as  $\mu$  increases, so also does the market bias toward over-investment in R&D and the optimal R&D subsidy rate  $s_R$  is a decreasing function of the R&D difficulty parameter  $\mu$ .

When  $\lambda$  is sufficiently large, that is, new products represent big improvements over existing products, then innovative firms are able to charge big markups of price over marginal cost and earn big profits. Under these circumstances, the negative business stealing and intertemporal spillover effects associated with R&D dominate the consumer surplus effect. Innovative firms do not take into account in their profit-maximizing calculations the large

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<sup>28</sup>In the limiting case where  $\mu = 0$ , R&D does not become more difficult over time and the intertemporal R&D spillover effect disappears. In Romer [1990], the intertemporal R&D spillover effect is positive because new designs are assumed to raise the productivity of all future individuals who do research.

losses in discounted profits incurred by the firms they drive out of business or the large R&D cost increases incurred by future firms. As a result, too large a fraction of the economy's resources are devoted to R&D and a R&D tax is welfare-maximizing. On the other hand, when  $\mu$  is small, that is, R&D only becomes slightly more difficult over time, then balanced growth for the economy is associated with innovations occurring frequently. As a result, innovative firms earn quality leader profits for a short period of time before they are driven out of business by further innovation. Under these circumstances, the positive consumer surplus effect dominates. Innovative firms only briefly benefit from their discoveries but the benefits to consumers last forever. Too small a fraction of the economy's resources are devoted to R&D and a R&D subsidy is welfare-maximizing.

Like in this paper, Jones [1995b] and Grossman and Helpman [1991a] also treat the size of innovations as exogenously given. Interestingly, both of these papers reach different conclusions concerning the optimality of R&D subsidies. We will now discuss the reasons why.

In Jones [1995b], the larger is the innovation size parameter  $\alpha$  (which determines the mark-up of price over marginal cost charged by producers of new intermediate inputs), the *larger* is the optimal R&D subsidy  $s_R$ . The main reason for this different conclusion is a different assumption about the nature of innovation. In Jones's model, new products are horizontally instead of vertically differentiated from older products. Because new products are different but not better, all innovative firms earn monopoly profits forever and no products ever become obsolete in Jones's model. The business stealing effect is much stronger in our model since innovative firms are able to drive competitors out of business.

In Grossman and Helpman [1991a], there is a n-shaped relationship between the innovation size parameter  $\lambda$  and the optimal R&D subsidy  $s_R$ : R&D taxes are optimal for very small or very large size innovations but R&D subsidies are optimal for intermediate-size innovations. The main reason for this more complicated relationship is that the equilibrium innovation rate is an increasing function of  $\lambda$  in their model. Since  $I$  appears in the denominator of the business stealing effect expression in (24), larger innovations lead to more firm turnover and this reduces the size of the business stealing effect. In our model, the equilibrium innovation rate only depends on the growth rate of the labor force and the

R&D difficulty parameter  $\mu$ . An increase in the size of innovations increases the relative size of the R&D sector but not the long-run innovation rate. Given that our model and Grossman and Helpman [1991a] share many assumptions, it appears that both the positive and normative properties of “quality ladder” endogenous growth models significantly change when less optimistic assumptions are made about the returns to R&D investment.

## 5 Concluding Comments

This paper presents a simple R&D-driven endogenous growth model to shed light on some puzzling trends in R&D employment levels, patent statistics and economic growth rates. In the model, firms engage in R&D to improve the quality of products, there is positive population growth and R&D becomes progressively more difficult over time. Equilibrium behavior involves gradual convergence to a balanced growth path in which both the share of total employment in R&D and the rate of patenting are constant over time. Thus the model can account for why patent statistics have been roughly constant even though R&D employment has risen sharply over the last 30 years. The model also illuminated why steadily increasing R&D effort has not lead to any upward trend in economic growth rates, as is predicted by earlier R&D-driven endogenous growth models with the “scale effect” property.

We find that three parameters (the innovation size parameter  $\lambda$ , the population growth rate  $n$ , and the R&D difficulty parameter  $\mu$ ) completely determine the long-run economic growth rate. A natural direction for further research is to endogenize each of these key parameters.

Preliminary analysis indicates that  $\lambda$  can be made endogenous by allowing firms to choose the size of their quality increments, along the same lines as in Grossman and Helpman [1991b, section 4.2]. Then public policies can be used to promote long-run economic growth by encouraging firms to search for larger improvements in product quality. Although R&D subsidies do not affect the equilibrium choice of innovation size by firms, the government can influence this choice by setting novelty requirements whereby patents are granted only to products that are sufficiently different from existing varieties. Changes in

the minimum novelty requirements have long-run growth effects when firms also choose innovation size.

The population growth rate  $n$  can be made endogenous by allowing parents to choose the number of their offspring and modelling the benefits and costs of having children. An interesting recent effort at modelling fertility choices and exploring the growth implications is Jones [1997b]. He assumes that people are altruistic: they benefit when their children enjoy higher levels of consumption and also benefit from having more children. On the other hand, there is a time cost to having children and wage income is sacrificed by having more children. Similar assumptions could be incorporated into the present model and the effects of public policies on fertility rates and economic growth rates could be explored. Surprisingly, Jones finds that R&D subsidies reduce the long-run rate of economic growth. It would be interesting to analyze the robustness of this conclusion by considering alternative ways of modelling fertility choices.

Although firms have some control over the size of the innovations they discover (some research projects are more ambitious in nature than others), and parents have some control over the number of children they produce, individual firms may not have any control over the rate at which R&D difficulty increases over time. We think of individual firms as choosing among an infinite array of R&D projects with varying degrees of expected difficulty. Firms start off pursuing the ex ante most promising projects and when success does not materialize, they gradually switch to exploring ex ante less promising projects. Individual firms do not control the rate at which R&D difficulty increases because they take the array of R&D projects as given as well as the industry-level rate at which firms move through the array, exploring different research directions. However, the government may be able to influence this rate through its funding of basic research. Basic research adds to the stock of ideas and can be thought of as adding new projects with differing degrees of expected difficulty to the infinite array of R&D projects firms choose from. Increased funding of basic research may reduce the rate at which R&D difficulty increases over time for firms doing applied research. Aghion and Howitt [1996] analyze a model with both basic and applied research and they study how the incentives to engage in both types of research jointly determine the steady-state rate of economic growth. Building on their analysis, an inter-

esting direction for further research is to add a basic research sector to the present model, where basic research increases the options available to applied researchers, and study how the equilibrium rate at which R&D difficulty increases over time differs from the optimal rate.

In addition to endogenizing key parameters that determine the long-run economic growth rate, the model could be developed further by introducing country heterogeneity. The endogenous growth model in this paper is a model of a *homogenous* world economy: workers in all countries are assumed to be equally productive and earn the same wage rate. Obviously, this is not realistic as there are large differences in the wages of workers in developed and developing countries. By introducing cross-country differences in labor productivity levels, one could analyze the circumstances under which the living standards in different countries converge over time. This represents yet another important direction for further research.<sup>29</sup>

## References

- Aghion, P. and Howitt, P. [1996], "Research and Development in the Growth Process," *Journal of Economic Growth*, 1, 49-73.
- Arrow, K. and Kurz, M. [1970], *Public Investment, the Rate of Return, and Optimal Fiscal Policy* (Baltimore: The Johns Hopkins Press).
- Baily, M. and Chakrabarti, A. [1985], "Innovation and Productivity in U.S. Industry," *Brookings Papers on Economic Activity*, 2, 609-639.
- Barro, R. and Sala-i-Martin, X. [1995], *Economic Growth* (New York: McGraw Hill).
- Cockburn, I. and Henderson, R. [1994], "Measuring Core Competence?: Evidence from the Pharmaceutical Industry," mimeo, MIT.
- Dinopoulos, E. and Segerstrom, P. [1996a], "The Dynamic Effects of Contingent Tariff

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<sup>29</sup>One trend that the model is not able to account for is the increase over time in the total number of patents granted by the U.S. government to foreigners (see Griliches [1994], p.10). A more complicated version of the model where foreign countries gradually "catch-up" with the U.S. may be able to account for this upward trend in foreigner patenting.

- Protection,” mimeo, Michigan State University.
- Dinopoulos, E. and Segerstrom, P. [1996b], “A Schumpeterian Model of Protection and Relative Wages,” mimeo, University of Florida.
- Evenson, R. [1984], “International Invention: Implications for Technology Market Analysis,” in *R&D, Patents and Productivity*, ed. Z. Griliches (Chicago: University of Chicago Press).
- Griliches, Z. [1990], “Patent Statistics as Economic Indicators: A Survey,” *Journal of Economic Literature*, 28, 1661-1707.
- Griliches, Z. [1994], “Productivity, R&D, and the Data Constraint,” *American Economic Review*, 84, 1-23.
- Grossman, G. and Helpman, E. [1991a], “Quality Ladders in the Theory of Growth,” *Review of Economic Studies*, 58, 43-61.
- Grossman, G. and Helpman, E. [1991b], *Innovation and Growth in the Global Economy* (Cambridge, MA: The MIT Press).
- Hall, B., Griliches, Z. and Hausman, J. [1986], “Patents and R&D: Is there a Lag?” *International Economic Review*, 27, 265-283.
- Jones, C. [1995a], “Time Series Tests of Endogenous Growth Models,” *Quarterly Journal of Economics*, 110, 495-525.
- Jones, C. [1995b], “R&D-Based Models of Economic Growth,” *Journal of Political Economy*, 103, 759-784.
- Jones, C. [1997a], “The Upcoming Slowdown in U.S. Economic Growth,” mimeo, Stanford University.
- Jones, C. [1997b], “Population and Ideas: A Theory of Endogenous Growth,” mimeo, Stanford University.
- Kremer, M. [1993], “Population Growth and Technological Change: One Million B.C. to 1990,” *Quarterly Journal of Economics*, 108, 681-716.
- Kortum, S. [1993], “Equilibrium R&D and the Decline in the Patent-R&D Ratio: U.S. Evidence,” *American Economic Review: Papers and Proceedings*, 83, 450-457.
- Kortum, S. [1996], “Research and Productivity Growth: Theory and Evidence from Patent Data,” National Bureau of Economic Research working paper No. 4646, Cambridge,

MA.

Machlup, F. [1962], *The Production and Distribution of Knowledge in the United States* (Princeton: Princeton University Press).

Mansfield, E. [1986], "Patents and Innovation: An Empirical Study," *Management Science*, 32, 173-181.

Malone, M. [1995], *The MicroProcessor: A Biography* (New York: Springer-Verlag).

McConville, D. [1994], "Intellectual Property Gains Respect: Patent Holders Have Never Had it so Good," *Industry Week*, 243, 33-38.

National Science Board [1993], *Science and Engineering Indicators-1993*. (Washington, D.C.: U.S. Government Printing Office).

Pakes, A. and Z. Griliches [1984], "Patents and R&D at the Firm Level: A First Look," in *R&D, Patents and Productivity*, ed. Z. Griliches (Chicago: University of Chicago Press).

Richardson, D. [1995], "Income Inequality and Trade: How to Think, What to Conclude," *Journal of Economic Perspectives*, 9, 33-55.

Romer, P. [1986], "Increasing Returns and Long-Run Growth," *Journal of Political Economy*, 94, 1002-1037.

Romer, P. [1990], "Endogenous Technological Change," *Journal of Political Economy*, 98, S71-S102.

Schankerman, M. and A. Pakes [1986], "Estimates of the Value of Patent Rights in European Countries During the Post-1950 Period," *The Economic Journal*, 96, 1052-1076.

Scherer, F. [1980], *Industrial Market Structure and Economic Performance* (Boston: Houghton-Mifflin).

Segerstrom, P., Anant, T., and Dinopoulos, E. [1990], "A Schumpeterian Model of the Product Life Cycle," *American Economic Review*, 80, 1077-1092.

WIPO [1983], *100 Years Protection of Industrial Property* (Geneva: World Intellectual Property Organization).

WIPO [annual issues], *Industrial Property Statistics* (Geneva: World Intellectual Property Organization).

Young, A. [1995], "Growth Without Scale Effects," National Bureau of Economic Research

## Appendix A: Welfare Maximization

In this appendix, we solve the social planner's optimal control problem (21).

The social planner's problem can be rewritten using  $x(t) = \frac{X_0 e^{\mu\Phi(t)}}{e^{nt}}$  instead of  $\Phi(t)$  as the state variable. This yields the equivalent optimal control problem

$$\max_{I(\cdot)} \int_0^\infty e^{-(\rho-n)t} \left\{ \frac{\log x(t) - \log X_0 + nt}{\mu} \log \lambda + \log \left[ 1 - \frac{I(t)x(t)}{A} \right] \right\} dt \quad (A1)$$

subject to the state equation  $\frac{\dot{x}(t)}{x(t)} = \mu I(t) - n$ , the initial condition  $[x(0) = X_0 > 0 \text{ given}]$ , and the control constraint  $[\frac{A}{x(t)} \geq I(t) \geq 0 \text{ for all } t]$ .

Ignoring the constant term  $\int_0^\infty e^{-(\rho-n)t} \left[ \frac{nt - \log X_0}{\mu} \right] \log \lambda dt$  which plays no role in determining the optimal control, the current value Hamiltonian function for the social planner's problem is given by

$$\mathcal{H} \equiv \frac{\log \lambda}{\mu} \log x + \log \left[ 1 - \frac{Ix}{A} \right] + \theta x [\mu I - n]. \quad (A2)$$

Solving for an interior solution ( $\frac{A}{x(t)} > I(t) > 0$  for all  $t$ ) yields the first order condition

$$\frac{\partial \mathcal{H}}{\partial I} = \theta x \mu - \frac{x}{A} \left[ 1 - \frac{Ix}{A} \right]^{-1} = 0, \quad (A3)$$

and the costate equation

$$\dot{\theta} = (\rho - n)\theta - \frac{\partial \mathcal{H}}{\partial x} = (\rho - n)\theta - \frac{\log \lambda}{\mu x} + \frac{I}{A} \left[ 1 - \frac{Ix}{A} \right]^{-1} - \theta(\mu I - n). \quad (A4)$$

Solving (A3) for  $I$  and substituting into (A4) and the state equation, we obtain a nonlinear, autonomous differential equation system:

$$\dot{\theta} = \rho\theta - \frac{\log \lambda}{\mu x}, \quad (A5)$$

$$\dot{x} = \mu A - \theta^{-1} - nx. \quad (A6)$$

The phase diagram corresponding to this system is illustrated in Figure 6. As illustrated, the  $\dot{x} = 0$  curve is upward sloping and the  $\dot{\theta} = 0$  curve is downward sloping. There is a

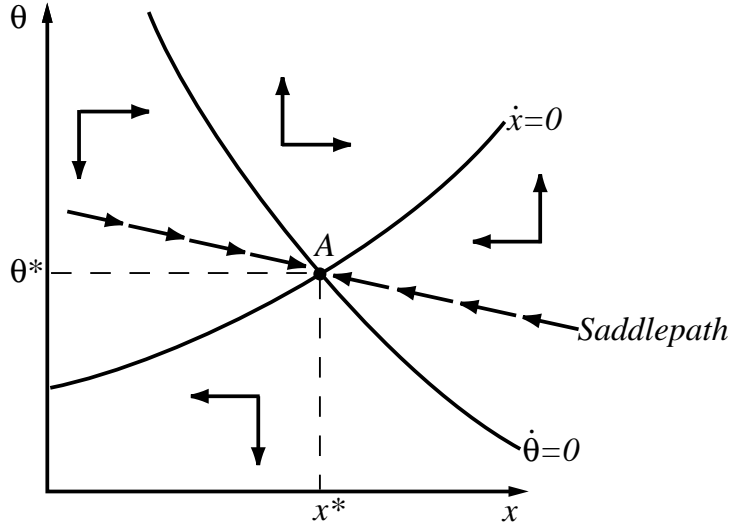


Figure 6: Stability of the optimal balanced growth path

unique steady state given by point  $A$  and this steady state is a saddle-point equilibrium of the differential equation system. Solving (A3) for  $I$  and substituting back into (A2), the “maximized Hamiltonian”  $\mathcal{H}^0 \equiv \frac{\log \lambda}{\mu} \log x + \mu A \theta - \log(A \theta \mu) - 1 - n \theta x$  is clearly a strictly concave function of  $x$  for given  $\theta$ . Thus by Proposition 10 in Arrow and Kurz [1970, p.51], jumping onto the saddlepath at time  $t = 0$  and staying on the saddlepath forever represents an optimal path for the economy.

Having established that there exists a unique optimal balanced growth path (given by point  $A$  in Figure 6) and that it is saddlepath stable, we now solve for the defining characteristics of this optimal balanced growth path. Substituting the steady state condition  $\dot{x} = 0$  back into the state equation yields the optimal balanced growth innovation rate

$$I = \frac{n}{\mu}. \quad (A7)$$

Solving  $\dot{x} = 0$  and  $\dot{\theta} = 0$  simultaneously using (A5) and (A6), we obtain  $x^* = A \left[ \frac{\rho}{\log \lambda} + \frac{n}{\mu} \right]^{-1}$ , which implies that the optimal balanced growth R&D ratio is

$$\frac{L_I}{L} = \frac{I x^*}{A} = \frac{1}{1 + \frac{\rho}{I \log \lambda}}. \quad (A8)$$

## Appendix B: Identifying the External Effects

In this appendix, we calculate the external benefits and costs associated with the marginal innovation. We use the same general procedure for identifying the external effects as in Grossman and Helpman [1991b, p.110-111].

We imagine that an external agent (e.g., a Martian) has achieved a single technological breakthrough, an improvement in quality in some industry  $\omega$  at time  $t = 0$ . We perturb the market equilibrium (with  $s_R = 0$ ) by  $d\Phi$  at each moment in time after  $t = 0$  (thereby preserving the initial path of innovation) and compute the impact on the welfare of agents other than the one who collects the profits from the marginal innovation. We ignore in this calculation the R&D costs incurred (and the profits earned) by the Martian innovator because, for firms that engage in R&D activities, private costs are exactly balanced by private benefits. Equation (8) implies that R&D firms earn zero expected discounted profits in equilibrium.

The effect of the extra product improvement on the welfare of agents other than the external one is found by differentiating (21) with respect to  $\Phi$ . Letting  $E(t) \equiv ce^{nt}$  denote aggregate consumer expenditure and recognizing that  $1 - \frac{IX_0 e^{\mu\Phi(t)}}{Ae^{nt}} = \frac{c}{\lambda} = \frac{E(t)}{\lambda e^{nt}}$ ,

$$\frac{dU}{d\Phi} = \int_0^\infty e^{-(\rho-n)t} \log \lambda dt + \int_0^\infty e^{-(\rho-n)t} \frac{1}{E(t)} \frac{dE(t)}{d\Phi(t)} dt. \quad (B1)$$

The first term on the RHS of (B1) is the marginal benefit at initial prices from consuming indefinitely a product of higher quality. This term represents the *consumer surplus effect*. The discounted value of this positive external effect is  $\frac{\log \lambda}{\rho-n}$ .

The second term on the RHS of (B1) reflects the effect of the new product on aggregate spending by agents other than the external one. Since these agents suffer a loss in profit income and the marginal innovation makes future innovations costlier to discover, their spending falls in response to the extra innovation. Therefore this term combines the *business stealing effect* and the *intertemporal R&D spillover effect*. We now calculate the discounted value of these two negative external effects.

Spending equals total income minus savings and savings equals investment in a closed economy. So  $E(t) = e^{nt} + \Pi(t) - \frac{IX_0 e^{\mu\Phi(t)}}{A}$ , where  $e^{nt}$  is wage income and  $\Pi(t)$  is aggregate profit income accruing to all innovative products other than the single, externally provided

one. Thus

$$\frac{dE(t)}{d\Phi(t)} = \frac{d\Pi(t)}{d\Phi(t)} - \frac{\mu I X_0 e^{\mu\Phi(t)}}{A}. \quad (B2)$$

The first term on the RHS of (B2) represents the business stealing effect and the second term represents the intertemporal R&D spillover effect. Consider now the loss in profits in the industry  $\omega$  where the “extra” innovation has taken place. In the event that there has been no further quality upgrade in that industry before time  $t$ , the economy forfeits profits equal to  $\frac{\lambda-1}{\lambda} E(t)$  at time  $t$ . Since the time duration of R&D races is exponentially distributed with parameter  $I$  and the probability that an innovation occurs by time  $t$  is  $1 - e^{-It}$ , these profits are forfeited with probability  $e^{-It}$  at time  $t$ . There will also be a multiplier effect since the profit shortfall in industry  $\omega$  induces a cut in aggregate spending and so a reduction in sales and income in other industries. The aggregate change in profit income, including the multiplier effect is

$$\frac{d\Pi(t)}{d\Phi(t)} = - \left[ \frac{\lambda-1}{\lambda} \right] E(t) e^{-It} + \left[ \frac{\lambda-1}{\lambda} \right] \frac{dE(t)}{d\Phi(t)}. \quad (B3)$$

Substituting (B3) back into (B2) and simplifying yields

$$\frac{1}{E(t)} \frac{dE(t)}{d\Phi(t)} = -(\lambda-1)e^{-It} - \frac{\mu I \lambda X_0 e^{\mu\Phi(t)}}{A E(t)}. \quad (B4)$$

Now we can substitute (B4) into the second term in (B1) and compute the integral. We find that the utility loss associated with the business stealing effect measures  $\frac{\lambda-1}{\rho+I-n}$ . Using (13) with  $s_R = 0$  and  $x(0) = X_0$ , the utility loss associated with the intertemporal R&D spillover effect measures  $\frac{\lambda I \mu X_0}{A c(\rho-n)} = \frac{\lambda-1}{\rho+I-n} \frac{n}{\rho-n}$ .

Summing all the external effects, (B1) becomes

$$\frac{dU}{d\Phi} = \frac{\log \lambda}{\rho-n} - \frac{\lambda-1}{\rho+I-n} \left[ 1 + \frac{n}{\rho-n} \right]. \quad (B5)$$

Consistent with (24), it is optimal to subsidize R&D if and only if  $\frac{dU}{d\Phi} > 0$ . The market rate of innovation is less than the optimal rate if and only if the consumer surplus effect outweighs the combined business stealing and intertemporal R&D spillover effects.